# Pieces of eight

Mike Mudge stimulates your little grey cells as he presents a combinatorial problem with application in computing. What can *you* do with eight ones and eight zeros?

The area of investigation this month has been suggested by W Johnson of Broom in south Yorkshire. It is concerned with the combination formed from eight ones and eight zeros. Of the 12,870 combinations of eight ones and eight zeros which are possible, it is the case that 256 have the following interesting, and useful, property.

Groups of four digits taken from a recurring listing in any 16 adjacent positions represent the numbers 0,1,2,...15. For example:

```
00001111011O0010100001111 etc
```

represents

```
0137FEDB6C925A48
```

There are actually only eight distinct combinations, each of which has 15 related combinations generated by rotation and another 16 generated by inverting the sequence and rotating. For example:

```
10100110111100001O100110 etc
```

represents

```
A4936DB7FEC80125
```

Problem 1 Design and implement a computer program to generate the eight distinct combinations referred to above. Verify that they are unique, and that the remaining 248 such combinations can indeed be generated as described.

The usefulness of the property lies in enabling an efficient binary-to-hexadecimal or hexadecimal-to-binary conversion routine to be constructed. The heart of this converter for binary-to-hexadecimal is:

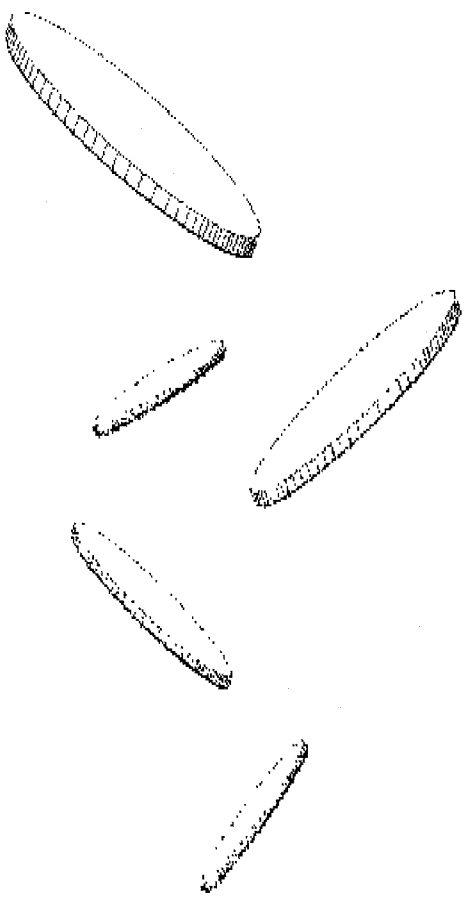
```
BINA$ = "100110100001011110: HEXA$ = "936DA480125B7FEC"
H$ = "": L = LEN(B$)
L = L + (4 - L MOD 4) MOD 4        'Make L a multiple of 4
B$ = RIGHT$ ("000" + B$, L)
FOR P = 1 TO L STEP 4
H$ = H$ = MID$ (HEXA$, INSTR(BINA$, MID$ (B$, P, 4)),1)
NEXT
```

Problem 2 Design and implement a Binary-to-Hexadecimal/Hexadecimal-to-Binary converter using the kernel suggested above, and compare its performance with any other such converter you may have access to.

Problem 3 Consider the possible extension of the above ideas to the design of general number base converters.

*Note:* Although such converters are of little interest in the world of computing, they constitute a valuable set of tools for some number theoretic investigations. See later 'Numbers Count' articles.

Attempts at some or all of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwlltrap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 April 1992. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* for the 'best' contribution arriving by the closing date.



It would be greatly appreciated if such submissions contained a brief description of the hardware used, program listings, run times and a summary of the results obtained, along with suggestions for further work in this area, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

## Review, July 1991: Aliquot Sequences and Safe Primes

This problem produced a very large response, most contributors concentrating on the Sequences of Safe Primes.

Nigel Backhouse 'laid the problem to rest' with a copy of 'Long Chains of Nearly Doubled Primes' by Günter Löh, *Mathematics of Computation*, vol 53, number 188, October 1989, pp751-759.

Chains up to length 13, *viz* k=12, were discovered in a search up to $2^{50}$ in Fortran 77 which ran for 520 CPU hours on a Siemens 7.882 computer.

Typical of many submissions, I mention Jim Duncan who found k = 7 starting with 1909919 in C on an Atari 1040 ST, and Paul Leyland who advanced to k = 8 starting at 85864769 in an hour or so on his Elonex 386B-2S, again programming in C.

However, after much soul searching, the very worthy prizewinner this month is Robin Merson of Farnham, Surrey. Robin, 'at last pushed to prepare multi-length routines for the Amstrad computer', used QuickBasic to find a longest sequence of k = 11 starting at 554688278429 and used a graphico-theoretic study to conjecture that for k greater than 6, the smallest starting value $q_1$ has $\log(q_1) \sim (27k - 77)/8$. *Viz* a sequence of length 20 is predicted with a 26-digit starting value.