The right angle

Mike Mudge presents a study of Right Angled Triangles , or Pythagoras Rules ABC!

The problem area this month is that of Right Angled Triangles (RATs) and the particular investigation has been suggested by Christopher John Roberts, alias CJ, of Twyford, Berkshire.

1) By inspection or otherwise define the necessary and sufficient conditions for a triangle with integer length sides to be a RAT. (Answer in the box.)

2) Construct and implement an algorithm to find the length, C, of the hypotenuse such that two RATs are possible. Give the first four values of C. (Answer in the box.)

3) Construct and implement an algorithm to find C such that four RATs are possible, giving the first four values.

4) Construct and implement an algorithm to determine how many, N(C), RATs are possible for a given value of C.

Test Data N(9125) = 10, N(32045) = 40, N(359125) = 52.

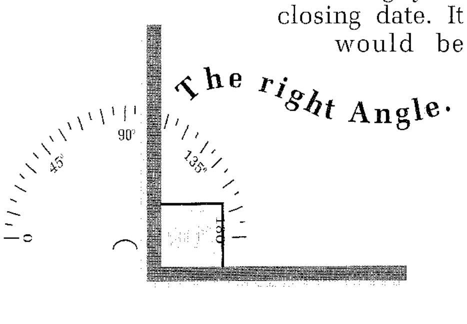
5) For the enthusiastic (all readers of Numbers Count, surely), construct and implement an algorithm to count RATs for all C-values, up to a specified C_{max}, and to list the associated A and B values in an orderly manner.

6) Due to Mike M. Extend the above concept to consider not C the hypotenuse of an integer-sided RAT, but D the body diagonal of a rectangular parallelepiped *viz* $D^2 = A^2 + B^2 + C^2$.

7) Due again to Mike M. What happens in n-dimensions to the regular hyper-parrallelepiped? Here the algebgra is the obvious generalisation, $X^2 = x^2_1 + x^2_2 + x^2_3 .....x^2_n$ where of course these x_i are all positive integers.

Suggest possible ARIT, RAT-like names for above 'shapes'.

Attempts at some or all of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 May 1992. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be greatly appreciated if such submissions contained a brief description of the hardware used, program listings, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*.



Answer 1) C has one prime factor, P_1, of the form 4*n + 1,for integer n.
Answer 2) C has a factor P^2_1, so 25, 169, 289 and 841 are the required values... the problem rapidly becomes more complicated!

Review, August 1991: a simple divisibility problem using Fibonacci-type sequences

This problem area produced an outstandingly large number of replies, all of which were of a very high standard. This seems an appropriate time to thank everyone who responded on this occasion and also all those who showed support for the 'Numbers Count' column during its temporary absence in September and October. Note that the column now has a full page.

Tiger Redman programmed in BASIC on a BBC B and reached k=6721 in about 100 seconds; HT Lovett-Turner (and others) noted that the 'Schofield Sequence' is identical to the original Henry Lucas Sequence (he also asks if Henry Lucas and Edouard Loucas are the same person?): he took k to 4181 in 7.5 hours in Borland Turbo BASIC on an Amstrad 1640HD20. Frank Webster advanced k to 15251 in 372 minutes in BBC BASIC on an Acorn Electron. A second submission from Tiger Redman advanced k to 29281, while Andrew Simpson went up to k = 35785 with a 7479 digit associated Lucas Number. Reference Fibonacci and Lucas Numbers (etc), S Vadja (1989).

Mr AGW Edmunds found the first 27 Schofield Pseudo-Primes, taking k up to 100127 in the ever popular BASIC on the equally popular BBC B. A contribution from PB Rayner used UBASIC86, a public domain version of BASIC which handles numbers up to 10^{2600}, on a Viglen Vig III PC with 20MHz 386DX processor and 387 co-processor together with 4Mb of memory. Paul listed the Schofield terms up to k = 10877 and then extended the search to the first 62 terms — that is, up to k= 497761 with an associated term(not printed) having over 100000 digits, the time being in excess of 16 hours.

Nigel Backhouse accompanied his study of this problem with a copy of *Mathematical Review* 49/8928 by A Peluso (New York) dealing with a most interesting-sounding paper, 'Some congruences of the Fibonacci numbers modulo prime p, by Hoggatt, VE Jr, and Bicknell, Marjorie, *Mathematical Magazine* vol 47 (1974) pp210-214. A sight of this work would be appreciated.

Reg Bond, a regular contributor from Derby, is to be congratulated on a four-pronged attack on this problem; the largest k value being 10525900321, the associated Schofield number having 2199783070 digits. However, if Paul Rayner's search is exhaustive (as I believe it to be) then unfortunately Reg has not found all the required terms, the first one missing being at k = 100065.

Finally, mention must be made of a computer-free contribution by E Knight-ing of Crowthorne, Berkshire, who constructs suitable k values as Carmichael numbers with all factors of the form5a ±1, thus 1.199.271 = 1024651, 31.61.211 = 399001, 31.61.271 = 512461, 31.61.631 = 1193221, 31.151.1171 = 5481451, 31.181.331 = 1857241 continuing up to 41.1721.35281 = 2489462641 with over $5x10^8$ digits in the associated term. EK also gave the theoretical verification of the correctness of these results.

The winner is Paul Rayner of Tonbridge, Kent. Congratulations, Paul.

Quick Calculation

Due to CJ: The End of the Diary Market?

The Editor, who could be described as careful with money, checked his desk and found an unused 1990 diary. Rather than put it in the bin he calculated when it would be valid again: that is, when all the dates and days matched. When is that? What is the answer for a 1989 or 1988 diary? Propose a simple rule for an old diary *i)* post 1963, *ii)* pre 1963.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.