A challenge in concentration

Mike Mudge considers the concentration rather than the separations of prime numbers.

The area of research this month has been suggested by Robin Merson of Farnham in Surrey. Robin is sure that some readers will turn up references to the works of Hardy/Litlewood, Staeckel and Lord Cherwell [*was this gentleman associated with the Lindemann electrometer?*] among others, and further asserts that 'anybody who has a go will learn something more about prime distributions'.

Prime Concentrations

Definition A regular prime k-set is a set of k prime numbers of the form $A(p).x+(n_1,n_2,n_3,....n_k)$, where A(p) is the product of primes up to p, and the width of the set $w(k) = n_k-n_1$, is as small as possible.

For k = 2 we have the prime pairs (6x - 1, 6x + 1) with w(2) = 2; for k = 3 the prime triplets (30x + 7, 11, 13), (30x + 11, 13, 17), (30x + 13, 17, 19) and (30x + 17, 19, 23) all with w(3) = 6. For k = 4 there is only one valid form, (30x + 11, 13, 17, 19) so the prime quadruplets have a width w(3) = 8.

For a given k there are usually one or two anomalous sets of small primes, eg (2, 3, 5, 7) forms a quadruplet but is not regular. The forms for small values of k are easily found. For sextets, w(6) = 16 and there is a single form 210x + (97,....113); while for octets, w(8) = 26, with four forms that can be written as 210x + (11,....37) and 210x + (17,....43).

Fig 1 F(k,N)

n \ k	4	6	7	8	9	10
10^3	4	1	1	2	3	1
10^4	11	1	2	3	4	
10^5	37	3	3	4	5	
10^6	165	3	7	7	8	
10^7	898	17	12	10	8	
10^8		81	33	19	8	
10^9		316	103	47	14	1
10^{10}						2
10^{11}						5

The first ones start with 11, 17, 1277, 88793, 113147.

The number of regular prime k-sets wholly less than n is denoted by F(k, n) and a tentative list of values for certain k and n is given in *Fig 1* below.

Problem 1
Prepare a program for finding, listing and/or counting regular prime k-sets, thus verify and extend as far as possible *Fig 1*.

For prime decads, w(10) = 32, with two firms 210x + (11,....,43) and 210x + (167,.......,199). So a prime decad will always have an imbedded octet and it should be a simple matter to check that there are no decads less than 10^9 except the trivial (11,....,43).

Problem 2
Find the first non-trivial decad (hint: it has 10-digit components) and the first non-trivial 11-set. [*What should this be called?*]

(Hint: A(23) can be held as a long integer and, if one writes the first member of a decad as $A(23).m + 210.s + n_1$, then for each m there are only 8190 possible values of s, which can be found and stored. This *avoidance* sieve can be followed by an *exclusion* sieve testing for division by primes greater than 23. For each value of m the number of possibles reduces to a mere handful.)

Fig 2 Additional check data

k	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
w(k)	2	6	8	12	16	20	26	30	32	36	42	48	50	56	?

The WIDTH function The width function, w(k), and the corresponding forms for k-sets are easily calculated up to k=18 using a list of the 480 numbers less than and prime to 2310. From this list it may appear that w(19) = 72 but unfortunately every possible such set has one member divisible by 13.

Problem 3
Find the values of w(k) for k≤18 together with the corresponding forms of the k-sets. Design and implement an algorithm for the cases k>18 and extend the w(k) values as far as possible.

The number of k-sets in an interval
It is 'well known' that the number of prime pairs (k=2) in an interval ΔN around N is given approximately by $C_2.\Delta N/(\log N)^2$, where the constant C_2 is given by the repeated product $2\prod_{p\geq3} p(p-2)/(p-1)^2 = 1.32...$ (An incidental computing exercise is provided by the evaluation of C_2 as accurately as possible.) More generally, the number of prime k- sets in an interval ΔN about N is of the form:

$F(k,N) = C_2.\Delta N/(\log N)^k$

where the C_k are constants. Formulae for the C_k can be found by elementary methods based upon the assumption that the distribution of primes may be treated as 'random'.

Problem 4
Derive formulae for the C_k and compare the calculated values of F(k, N) with the observed values.

Problem 5
In the prime desert beyond say 10^{100} there will undoubtedly be cases of 100 or more primes. How many digits would you expect in the smallest regular set of primes with k=100?

Robin wishes to reassure readers by stating that 'It should take no more than a few hours to find a 30-digit quadruplet or a 15-digit octet. So there is plenty of scope for many weeks' enjoyable computing!'

See *Fig 2* for additional check data.

Attempts at some or all of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 July 1992.

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.