Going by BUSS

An investigation of Unique Summation Sequences, presented by Mike Mudge.

The following exposition was suggested in part by the Master of Arts thesis submitted by Peter N Muller at the State University of New York in February 1967.

Some details of the BUSS

k	1	2	3	4	5...	10...	50...	500...	5000...	20000...
a_k	1	2	3	4	6...	18...	253...	5685...	65217...	268553...

Definitions

(i) A Summation Sequence, (a_k), is an increasing sequence of positive integers for which a_1 and a_2 are arbitrary and such that for every $k>2$, a_k can be expressed *uniquely* as the sum of two *distinct* elements of the sequence.

(ii) The pair (a_1,a_2) is called The Base of the Summation Sequence (SS).

(iii) A Unique Summation Sequence (USS) is a summation sequence such that for any $k>2$, a_k is an element of the sequence *if and only if* it can be expressed uniquely as the sum of two *distinct* elements of the sequence.

Note The BASE defines the USS completely and in particular the BASE (1,2) determines the USS called the BIADD sequence. We shall restrict our discussion to the BIADD sequence (BUSS).

(iv) An integer which is an element of the BUSS is called a BIADD.

(v) An integer which cannot be expressed as a sum of two BIADDS is called a BINULL.

(vi) An integer, n, which can be expressed as the sum of two BIADDS in more than one way is said to be multiply expressible. The number of ways in which this can be achieved is called the degree of expressibility of n, and is denoted by E(n).

(vii) Each BIADD (except 1 and 2) determines (and is determined by) a unique pair of BIADDS known as its lower and upper components.

Note A BIADD can occur as an upper component in only a finite number of pairings, but there appears to be no limit to the number of pairings for which it is the lower component (see above).

Observe that a_5 is *not* equal to 5 as this can be constructed in more than one way *viz* 4+1 & 3+2 similarly a_6 + 8, a_7 + 11 etc. If B_n denotes the set of the first n BIADDS, we follow Muller in denoting by $\bar{B}$ the set B_{20000} with greatest member 268553.

Consider now the frequency of occurrence of each BIADD as a lower component. Theoretically the BIADDS in $\bar{B}$ could have 19998 distinct lower components. (No components are defined for the BASE terms 1&2.) However, in $\bar{B}$, only three BIADDS *viz* 3, 4, and 48 have a lower component of 3.

Conjecture 1 There are no pairs of consecutive BIADDS greater than a_{16} + 47 and a_{17} + 48.

Problem 1 Design and implement an algorithm to compute the terms of the BUSS together with their upper and lower components. Hence tabulate E(k) as a function of k, tabulate the frequency of occurrence of each BIADD as a lower component and discuss the content of Conjecture 1.

Conjecture 2 a_n, the n^{th} term in the BUSS, is asymptotically (that is, for larger and larger n) approximated to by n log (n).

Problem 2 Compare both a_n - n log (n) and a_n/ (n log (n)) as a function of n and hence investigate the content of conjecture 2 for n as large as possible.

Definition (viii) A(x) denotes the number of BIADDS not greater than x, while N(x) denotes the number of BINULLS (see definition (v) above) not greater than x.

Conjecture 3 N(x)/P(x) is asymptotically equal to a constant.

Problem 3 Compute N(x)/P(x) as a function of x and either propose a numerical value for c or supply reasons to refute conjecture 3.

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 August 1992. It would be appreciated if the submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in the area of Unique Summation Series (particularly with a view to a generalisation of the concept of using pairs as generators!) all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if an SAE is provided.

Review, January 1992: Rising and falling with Hailstone Numbers

Roy Sayers of Leeds used an Elonex 386 with GW-BASIC, concentrating his efforts on Problem C and covering a wide range of parameters. However, he experienced considerable difficulty with rounding errors and is interested in examining 5x+5 with starting values x_0 = 13,17 & 21 with high-precision integer arithmetic. Any offers of help here?

The very worthy prizewinner this month is Ed Hersom of Thirsk, North Yorkshire, using an RML Nimbus 8086 with its 8087 numeric processor. Ed experienced no loss of accuracy up to 10^{15}, saying he 'used this as my safety limit and, as it turned out, this did not limit my work in any way'. Ed supplied fully annoted Forth programs, and among his innovative ideas the approach to Problem C deserves detailed mention. Considering the generalisation a*x+b, it was felt that (EH) 'a' other than 3 was a game in a different ball park and so it was ignored; however, with b and x_0 running from 1 to 37 the loop structure was investigated. If n denotes the number of steps in a loop, then the parameter n/b referred to as The Figure of Merit was found never to exceed 3. Does this have a theoretical significance?

Ed also generated trajectories running backwards directly. This algorithm found all the starting values of x (below some specified x_{max}) whose trajectories do not exceed some arbitrary limit. More details from EH or MM.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.