The Smarandache Function

The Smarandache Function, together with a sample of The Infinity of Unsolved Problems associated with it, presented by Mike Mudge.

The Smarandache Function, S(n) (originated by Florentin Smarandache — *Smarandache Function Journal*, vol 1, no 1, December 1990. ISSN 1053-4792) is defined for all non-null integers, n, to be the smallest integer such that (S(n))! is divisible by n.

Note N! denotes the factorial function, N!=1x2x3x...xN: for all positive integer N. In addition 0! = 1 by definition.

S(n) is an even function. That is, S(n) = S(-n) since if (S(n))! is divisible by n it is also divisible by -n.

S(p) = p when p is a prime number, since no factorial less than p! has a factor p in this case where p is prime.

The values of S(n) in *Fig 1* are easily verified. For example, S(14) = 7 because 7 is the smallest number such that 7! is divisible by 14.

Problem (i) Design and implement an algorithm to generate and store/tabulate S(n) as a function of n.

Hint It may be advantageous to consider the STANDARD FORM of n, *viz* n $= ep_1^{a_1}p_2^{a_2}......p_r^{a_r}$ where e = ± 1, $p_1, p_2, ... p_r$ denote the distinct prime factors of n and $a_1, a_2, ...a_r$ are their respective multiplicities.

Problem (ii) Investigate those sets of consecutive integers (i,i+1,i+2,...i+x) for which S generates a monotonic increasing (or indeed monotonic decreasing) sequence.

Note For (1,2,3,4,5) S generates the monotonic increasing sequence 0,2,3,4,5; here i = 1 & x = 4.

If possible estimate the largest value of x.

Problem (iii) Investigate the existence of integers m,n,p,q & k with n≠m and p≠q for which:
either **(A)**: S(m) + S(m+1) +....S(m+p) = S(n) + S(m+1) +....S(n+q) or **(B)**:

$$\frac{S(m)^2 + S(m+1)^2 +....S(m+p)^2}{S(n)^2 + S(n+1)^2 +....S(n+p)^2} = k$$

Problem (iv) Find the smallest integer k for which it is true that <u>for all n less than some given n_0</u> at least one of:
S(n), S(n+1)....S(n+k-1) is:
A) a perfect square
B) a divisor of k^n
C) a factorial of a positive integer.

Conjecture what happens to k as n_0 tends to infinity: i.e. becomes larger and larger.

Problem (v) <u>Construct prime numbers of the form</u> $\overline{S(n)\,S(n+1)...S(n+k)}$: where $\overline{abcdefg}$ denotes the integer formed by the concatenation of a,b,c,d,e,f & g. For example, trivially $\overline{S(2)\,S(3)}$ = 23 which <u>is prime, but no so trivially</u> $\overline{S(14)S(15)S(16)S(17)}$ = 75617, also prime!

Definition An A-SEQUENCE is an integer sequence $a_1, a_2, ...$ with $1 \le a_1 < a_2 < ...$ such that no a_i is the sum of distinct members of the sequence (other than a_i).

Problem (vi) Investigate the construction of A-SEQUENCES $a_1, a_2, ...$ such that the associated sequences $S(a_1), S(a_2), ...$ are also A-SEQUENCES.

Definition The k^{th} order forward finite differences of the Smarandache function are defined thus:

$D_S(x)$ = *modulus(S(x+1) - S(x)),
$D_S^{(k)}(x) = D(D(...k\text{-times}D_S(x)...))$

Problem (vii) Investigate the conjecture that $D_S^{(k)}(1)$ = 1 or 0 for all k greater than or equal to 2.

c.f. Gilbreath's conjecture on prime numbers, discussed in 'Numbers Count' *PCW* Dec 1983. * Here modulus is taken to mean the absolute value of (ABS.), modulus (y) = y if y is positive and modulus (y) = -y if y is negative.

The following selection of Diophantine Equations (i.e. solutions are sought in integer values of x) are taken from the Smarandache Journal and make up:

Problem (viii) If m & n are given integers, solve each of:
a) S(x) = S(x+1), conjectured to have no solution
b) S(mx+n) = x
c) S(mx+n) = m+nx
d) S(mx+n) = x!
e) $S(x^m) = x^n$
f) $S(x)^m = S(x^n)$
g) S(x) + y = x + S(y), x&y not prime
h) S(x) + S(y) = S (x+y)
i) S(x+y) = S(x)S(y)
j) S(xy) = S(x)S(y)

Attempts at some or all of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 September 1992.

Review, February 1992: Pieces of Eight and a Binary/Hexadecimal Converter

This problem, originated by W Johnson of Rotherham, considered the 12,870 combinations of eight ones and eight zeros and requested the eight distinct combinations each of which yielded 15 related combinations by rotation and another 16 by inversion and then rotation.

The prizewinner this month is Richard Tindall of Great Shelford, Cambridge, who actually found the required eight solutions using simple logic: 'I find it a waste of time to use a computer...'. Richard then discussed the efficient program for binary/hexadecimal conversion in terms of both Assembler and Algol 68. A most interesting piece of work. Well done!

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.

Fig 1

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S(n)	0	2	3	4	5	3	7	4	6	5	11	4	13	7	5