This month try tackling The Kaprekar Process, an iterative scheme involving Digit Shuffling, presented by our regular master of mathematics, Mike Mudge.

Notation ${}^sN_r$ represents any positive integer containing s - digits. Associated with it are two other positive integers ${}^s\overline{N}_r$ and ${}^s\underline{N}_r$, these are obtained by arranging the digits of the original integer in ascending and descending orders of magnitude respectively.

The *Kaprekar Iteration Process is defined by:

$${}^sN_{r+1} = F({}^sN_r) = {}^s\overline{N}_r - {}^s\underline{N}_r$$

A Result *DR Kaprekar of the Indian Institute of Science in Bangalore published the following result:

If s = 4, with certain exceptions the iterative process described above ultimately reaches ${}^4N = 6174$; for which $\overline{N}(6174) = 7641$ while $\underline{N}(6174) = 1467$ and hence F(6174) = 7641 - 1467 = 6174.

Problem 1 Verify the above result of DR Kaprekar, identifying carefully the 'certain exceptions'.

Problem 2 Extend Kaprekar's result to the cases s = 5,6,7,8.... (Trivially, when s = 2 the loop 63 - 27-45 - 09 - 63 is reached, and when s = 3 the value 495 is reached.)

Problem 3 How do the above results appear when the arithmetic is carried out in radix different from ten? In particular, binary and hexadecimal arithmetic are worthy of further study.

Definition 6174 is said to be a *Harshad Number*, by which is meant that it is exactly divisible by the sum of its digits. (6+1+7+4) * 343 = 6174. Among other Harshad Numbers are 1729 = (1+7+2+9) * 91; the famous taxi registration in the Hardy/Ramanujan anecdote. For further study of these fascinating numbers readers are referred to Numbers Count in *PCW* August 1983 and January 1984.

Problem 4 Are any other Harshad Numbers ultimately reached by the Kaprekar Iterative Scheme?

Now for something very strange...

Problem 5 Instead of ${}^s\overline{N}$ and ${}^s\underline{N}$ consider the digits ordered in alphabetical and reverse alphabetical order. Define F* as the modulus of the difference between these two numbers and investigate the iteration of F*.

Note The result of applying F* to a given integer depends upon the language being used (see below).

In English	F*(6174) = mod(4176 - 6714 = 2538
In French	F*(6174) = mod(4761 - 1674) = 3087
In German	F*(6174) = mod(1674 - 4761) = 3087
In Welsh	F*(6174) = mod(6741 - 1746) = 4725

Note The replacement of an integer by the associated ordered (or unordered) digit string yields many fascintating areas of research in addition to the Kaprekar Iteration.

Among the most interesting I believe to be the search for sets of *Powerful Numbers* of *Degree* — k; these are defined to be those positive integers which are numerically equal to the sum of the k — th powers of their constituent digits. (Radix 10, or as a separate study of any other radix which appeals.) Thus when k = 7 the number 14459929 is *Powerful*;

$$(14459929=1^7+4^7+4^7+5^7+9^7+9^7+2^7+9^7)$$

similarly for k = 8,88593477 and for k = 9 the number 912985153 are members of the set of *Powerful Numbers*.

Problem 6 Find the complete sets (P_k) of *Powerful Numbers* for k = 1,2,3,4,5....

Attempts at some, or all, of the above problems maybe sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 December 1992. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded, by *PCW*, to the 'best' contribution arriving by the closing date.

Additionally, the author would be interested in general or specific comments or suggestions relating to problems involving the study of integers in terms of the properties associated with their digit strings... such as that of powerful numbers referred to earlier. Is this genuine number theory or is it too artificial?

Review, April 1992: The Problem with Plates, a study in geometrical dissection

This represented the first serious attempt to interest Numbers Count readers in geometrical problems and was inspired by the thesis of AJW Duijvestijn at the Tech.Hoge-school in Eindhoven, 1962, entitled 'Electronic Computation of Squared Rectangles'. The later results are contained in AJWD, 'An algorithm that investigates the planarity of a network', *Computers in Mathematical Research*, North-Holland, Amsterdam 1968; and AJWD, 'Simple perfect squared square of lowest order', J Combin Theory, Ser B 25 (1978) pp240-243. The subsequent discussion of Mrs Perkins' Quilt was spoilt by printers' distortion of the f(n) table which should appear as shown in *Fig 1*.

The most popular answers to M(i) & M(ii) are 'probably not' and to M(iii) 'probably yes'! See Z Moron, 'On the dissection of rectangles into squares', Wiadom Mat (2)i(1955) pp75-94 and also 'On almost perfect decompositions of rectangles', Wiadom Mat (2)i(1955/56) pp175-179.

Mention must be made of a superb effort by Robin Merson, obtaining the table in *Fig 1* in almost every detail and extending to f(n)=24 in part. However, after some thought the prizewinner this month is Michael Meierruth of Via Treviso 33, 20127 Milan, Italy, for submissions including the deduction and drawing of the 32x33 solution in 20 secs, and the 47x65 solution in several hours on a Compaq 386/20e. Michael found references to the computer construction of the 112x112 dissection of order 21 but felt that a deeper knowledge of the problem was required to generate this. The above references may help with this task.

f(n)	1	4	6	7	8	9	10	11	12
n	1	2	3	4	5	6,7	8,9	10-13	14-17

f(n)	13	14	15	16	17	18
n	18-23	24-29	30-39,41	40,42-50	51-66, 68,70	67,69, 71-87

f(n)	19
n	88-100

Fig 1

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.