Tetrahedra with integer edges, and other interesting solids, presented by Mike Mudge.

This area of investigation has been suggested by David Broughton of Freshwater, Isle of Wight. It has no hidden mathematical theory and gives readers an opportunity, not frequently available in 'Numbers Count', to use computer graphics as well as ingenuity in the design and implementation of both logical and arithmetic routines.

Recall that three given lengths do not necessarily form a triangle: any length having to be less than the sum of the remaining two for the triangle to exist. Now, given four face-triangles having the necessary common edges, a tetrahedron is not necessarily defined. For example, consider the four triangles whose integer length sides are (10,12,3), (10,13,9), (13,11,3) and (12,11,9) respectively.

**A tetrahedron is a* polyhedron *(solid three-dimensional geometrical figure) having four (necessarily triangular) faces. Furthermore, when a tetrahedron is formed from six given edges it may not be unique, even when the mirror image of the initial shape is considered indistinguishable from that shape. David Broughton suggests that there may be as many as 30 distinct tetrahedra defined by a given sextuplet of edge lengths.*

Notation The tetrahedron a, b, c, d, e, f: has one face formed from a, b & c, then d, e & f are the 'opposite' edges to a, b & c respectively, where 'opposite' is defined as having *no common vortex.* Hence the four triangular faces of a, b, c, d, e, f have sides a, b, c; c, d, e; b, d, f and f, e, a.

Note regarding symmetry If a, b, c, d, e and f are all different lengths, then there are 24 different permutations of these six edges which specify the *same* tetrahedron.

David Broughton defines a *canonical ordering* (for the case where all the edges have different lengths) as follows: the first edge is to be the largest (longest), the second is the largest of the four which share a vertex with the first, the third is the edge which makes the third side of the triangle with the first two, and the ordering of the remaining three follows from the above notation.

A proper zero-volume tetrahedron is one which has zero volume but does not have zero-area triangles for its faces!

Those readers with a background which will permit them to evaluate a third order determinant are encouraged to read the brief note entitled 'Volume of a Tetrahedron' by Frank Chorlton in the *Mathematical Gazette,* vol 75, no 472, June 1991, pp196-197, where a most elegant formula is derived (from the triple scalar product so useful in vector algebra) giving the volume of a tetrahedron as a function of the six edge lengths. However, it must be stressed that an understanding of, or access to, this result is not needed to conduct the following investigation.

Question 1 What is the smallest consecutive set of *prime numbers* that defines the edge lengths of a tetrahedron?

Question 2 Which proper zero-volume tetrahedron has the smallest maximum edge length? It is assumed throughout this article that all edge lengths are to be positive integers, except Question 5 below.

Question 3 What is the smallest non-zero volume of a tetrahedron with six distinct integer edge lengths?

Question 4 What is the smallest integer volume of a tetrahedron with six distinct integer edge lengths?

Question 5 What is the smallest volume of a tetrahedron having four distinct integer face areas?

Question 6 For those readers whose computers are not operational at the present time! Derive a formula for the volume of a tetrahedron as an algebraic function of its edge lengths.

General Extension Consider the three regular polyhedra known as Octahedra, Dodecahedra and Icosahedra with 8, 12 and 20 faces which are equilateral triangles, regular pentagons and equilateral triangles respectively. Investigate the possibilities of integer edges, faces and/or volumes.

If the concept of regularity, i.e. all faces/edges/vertices being identical, is dropped, it is clear that this analysis becomes completely intractable... I think!

Attempts at some or all of the above questions may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ (tel 0994 231121), to arrive by 1 January 1993. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. The author would also be interested in any general or specific comments relating to the study of three (or higher) dimensional figures having integer values for angles, lengths, areas, volumes etc.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review: May 1992
A Challenge in Concentration, or 'Where have all the Prime Numbers Gone?'

This research area was suggested by Robin Merson of Farnham in Surrey. In addition to all the results due to Robin, it is appropriate to mention the paper, 'Large Intervals between Consecutive Primes' by JH Cadwell, *Mathematics of Computation* vol 25, no 116, Oct 1971, pp909-913. The latter uses statistical arguments to obtain a formula for the largest interval between consecutive primes, which agrees well with recorded values up to 10^{15}. The whole topic of Heuristic and Probabilistic Results about Prime Numbers is updated as Chapter 6 in Paulo Ribenboim's book *Prime Number Records.*

This month's prizewinner, however, is Gareth Suggett of 34 Bridge Road, Worthing, Susssex, BN14, 7BX.

Gareth, in addition to questioning the uniqueness of A(p), i.e. why for k = 6 use 210x + (97, 101, 103, 107, 109, 113) instead of 30x + (7,11,13,17,19,23)? and verifying the quoted results, also found three sets for k = 6 between 10^4 and 10^5, not two as quoted. These are as shown below:

Well done, Gareth!

16057, 16061, 16063, 16067, 16069, 16073;
19417, 19421, 19423, 19427, 19429, 19433;
43777, 43781, 43783, 43787, 43789, 43793.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.