A 'train' of thoughts from the Western Railway, Jaipur, India, related by Mike Mudge.

The triad of problems this month, although associated with different areas of empirical number theory, share a commonality of origin. Shyam Sunder Gupta, a chief engineer with the Western Railway in Jaipur, India, has recently published results on them and he has offered them to Numbers Count. Each provides the enthusiastic amateur with the opportunity to obtain previously unknown results, which may in turn assist in the discovery of underlying mathematical theory. No background in advanced mathematics is required, although the scope for ingenious programming is virtually unlimited.

DIGIT PRODUCT TERMINI (DPT)

The DPT of a positive integer is obtained by iterating the process of forming the product of its constituent digits until a single digit is obtained. (Compare and contrast this algorithm with the construction of the Digital Root, obtained by iterating the process of forming the sum of the constituent digits until a single digit is obtained.) Among the questions related to DPT and posed by SS Gupta are:

(i) To find two or more consecutive numbers with the same DPT, for example 441..16..6; 442..32..6; 443..48..32..6.

(ii) To find the relative frequencies of numbers having DPT of 2, 4, 5, 6, 8 and 0. 'As numbers with DPT of 1, 3, 7 and 9 have only a specific form, their relative frequencies are known?'

(iii) To investigate the DPT generated by special numbers such as figurate, prime and Fibonacci.

Is there any connection, in general, between DPT and PERSISTENCE? (See Numbers Count, November 1983.)

RARE NUMBERS

These are defined as numbers which generate a perfect square when added to their reverse and also when their reverse is subtracted from them. For example, 65 is a Rare Number because $65 + 56 = 121 = 11^2$ and $65 - 56 = 9 = 3^2$.

There are only five known non-palindromic Rare Numbers which are less than 10^{10}: see SS Gupta, *Journal '2001'*, June 1990, p77.

(i) Find the five such numbers less than 10^{10}, extend this sequence and conjecture whether the number of Rare Numbers is finite or infinite.

(ii) Investigate the nature of Palindromic Rare Numbers.

(iii) Consider the existence of Rare Numbers in number systems other than Base Ten. An obvious choice would seem to be Hexadecimal; another could possibly be Octal.

(iv) What about Very Rare Numbers which generate two cubes or even higher powers by having their reverse added to and subtracted from them?

(v) Do any Hybrid Rare Numbers exist which yield, for example, one square and one cube, or perhaps a fifth and fourth power?

EQUAL PRODUCT OF REVERSIBLE NUMBERS (EPORNS)

These are defined as integers which can be expressed as a product of two reversible numbers in two different ways. Thus 4030 = 130 x 031 = 310 x 013, and 16746912 = 2556 x 6552 = 4473 x 3744. (See SS Gupta, *Science Today*, February 1987, p77.)

Verify that all four-digit EPORNS are multiples of 10 and that all five-digit EPORNS except 63504 are also multiples of 10.

Investigate the frequency of occurrence of EPORNS and prove that an EPORN is a perfect square only when it is the square of a palindrome. Is it possible for an EPORN to be a cube or higher power?

Attempts at some or all of the above investigations may be sent to: Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 February 1993. Any communications received will be judged using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if submissions contain a brief description of the hardware used, details of programs, run times and a summary of the results obtained, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is included.

I would also be interested in any general or specific comments relating to the three topics outlined this month and will undertake to pass them on to SS Gupta in return for his encouragement to use them in Numbers Count.

Review, June 1992:
'Going by BUSS', an investigation of Unique Summation Sequences

This contribution generated a number of extended and well-presented responses, most encouraging for a time of year when gardening and holidays often take priority over 'fun' computing. However, it also contained misprints, showing the lower component of 3, 4 and 48 as 3 rather than 1, and replacing A(x) with P(x) in Problem 3 and = with + in Conjecture 1...

The following analysis of run time by Ed Hersom of Thirsk deserves specific mention. It relates to an RML Nimbus, an 80186, running at 8MHz with an 8087 for doing 'hard' sums. Ed conjectured, on the basis of initial runs, that his algorithm followed the law:

Log (t) = 1.86 (Log(n) - 2.86), where t is the computing time in seconds and n is the maximum integer that has been examined in search for a BIADD. This predicted 16 hours and 40 minutes for the value 268553 quoted by Muller, but Ed generated this result in a pleasing nine hours. Furthermore, he was able to explain this improved time in terms of his algorithm (written in FORTH).

Gerhard Lantzsch and his son, of Berlin, used a no-name computer with an Intel 486 running at 33MHz. Gerhard struggled but was not deterred by the misprints, and has proposed many possible ideas for future Numbers Count articles. I am certain that readers will hear more of him in the future.

This month's clear winner is Hugh Spence of 22 Orissa Road, Plumstead, London SE18 1RG. Hugh used a 25MHz 386SX generic (DSi motherboard and case) running DRDOS 6.0 and programming in TopSpeed Modula 2 and the compact memory model. All three problems were answered up to 60,000 (three times Miller's attempt) in 46 hours of computing time. A superb effort, culminating in a guess at the limit in Problem 3 of 'around 2.52'. Any other offers of data support (or theory)?

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.