Mike Mudge wonders what 'The Game of Life' has in store for *PCW* readers.

Many readers are probably familiar with the 'Game of Life' as proposed by Professor John Conway, *Scientific American*, October 1970. The game consists of an arbitrarily large grid of square cells, where each cell is regarded as either alive or dead, conventionally shaded (or having a symbol inside it) if alive, and blank (or empty) if dead. The game is played as follows: start with a first generation pattern of living cells, which may be regular, random or a mixture of both, and either finite or (in theory only) infinite. To obtain the next generation, apply the following rules concurrently to each cell, C, on the grid. If C is living and touches two or three living cells it remains alive, otherwise it dies. If C is dead and it touches exactly three living cells it becomes alive, otherwise it remains dead. Repeat this process for as many generations as are of interest. A simple initial investigation using pencil and paper will reveal a host of stable shapes such as a block of four live cells, together with simple two-state oscillators such as three live cells in a row (a blinker). There are also shapes that move across the grid as each generation arises. The most common of these is called a 'glider'.

Algebraic formulation of the above rules

Define ENVIRONMENT, E, as the number of living neighbours required to prevent a currently living cell from expiring. $E_1 \leq E \leq E_u$. Similarly FERTILITY, F, is to denote the number of living neighbours required to bring a dead cell to life. $F_1 \leq F \leq F_u$. The TRANSITION RULE, R, defining the game, is simply the ordered quartet (E_1, E_u, F_1, F_u).

For Conway's Game of Life R = (2,3,3,3), but other related games are available to us; R = (3,4,3,4) has been extensively studied and exhibits a variety of oscillators that are totaly different from those in R = (2,3,3,3). Unfortunately it is easy in "3-4" life to produce starting patterns that rapidly expand forever. While this is possible in Conway's Life, it does require careful consideration of the first generation for this to occur.

Extensions to this two-dimensional life have involved "ageing" over a number of generations, that is greyscales rather than dead/alive and also the use of hexagonal cells, but we are not considering these at this time.

Now, what do we seek from Three Dimensional Life? For the following discussion I acknowledge a communication from Carter Bays, Dept of Computer Science, University of South Carolina, Columbia, SC 29208, USA, entitled The Game of Three Dimensional Life and dated 20/11/86. In three dimensions a cell can have from zero to 26 living neighbours; hence we may construct a large variety of rules. Specificallly we can have $(E_1, E_u) = (1,1)$; (1,2);...(1;26); (2,2)... a total of 351 ordered pairs; similarly for (F_1, F_u), yielding 351 x 351 = 123201 possible rules. Fortunately most of these generate uninteresting forms which either expand rapidly and indefinitely or quickly shrink and disappear.

It is "easy to see" that if $F_1 \leq 4$, indefinite expansion will occur while $F_1 \geq 10$ will lead to a bounded form although the interior may remain in turmoil.

Empirical study shows that rules dealing with four to seven neighbours have the greatest potential interest and of those investigated by Carter Bays, R = (4,5,5,5) and R = (5,7,6,6) seem to be the most interesting. The latter leads to stable and oscillating forms that are similar in many ways to those of Conway's (2,3,3,3) in two dimensions. The former may ultimately prove to be the more rewarding rule since it requires more time to 'settle down' and hence there is more opportunity for interesting intermediate reactions. There exists also an abundance of small stable and oscillating forms that almost always exhibit some sort of symmetry; for example, a 2x2x2 cube with two side-adjacent elements deleted, called the stable ""V"".

PROJECT A Design and implement an algorithm to play generalised two dimensional life with input rule R = E_1, E_u, F_1, F_u), a given size of universe, and edge conditions left to the imagination of individual contributors. Investigate in particular Conway's game using R = (2,3,3,3) and an 'infinite' universe. Catalogue, with suitable names, interesting objects which arise.

PROJECT B Repeat for three dimensional life with particular attention being given to Bays rules R = (4,5,5,5) and R = (5,7,6,6).

Note An interesting test case involving an initially 'random' distribution of living cells throughout the universe allowed to evolve for some 60 or 70 generations can be expected to exhibit many stable or quasi-stable (oscillatory) objects.

Attempts at either or both of the above projects may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 March 1993. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

Review, July 1992
The Smarandache Function: a first visit?

This topic is certain to be revisited in the near future, and the lack of space available here will certainly be remedied on that occasion. Suffice it to report that Jim Duncan computed up to S(1499999), the first million taking 50 hours in Lattice C on an Atari 1040ST. In Problem (ii), no evidence for a largest value of x was found, while in Problem (vii) the conjecture was verified for the first 32,000 values of S(n). The very worthy prizewinner is John McCarthy of 17 Mount Street, Mansfield, Notts NG19 7AT, who has extensively *investigated* the computation of S(n) up to 2^{32}; arriving at conclusions such as: 'several years of computing', 'at least 12Gb of disk space' and '3,303,302 pages of output'. John's concluding comment, 'Am I mad?', is clearly answered NO! by examining his specimen pages of output including those relating to 10-digit values of n. Listings supplied. Details from John directly upon request.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.