Mike Mudge pays a return visit to the Florentin Smarandache Function.

The originator of this function, Florentin Smarandache, an Eastern European mathematician, escaped from the country of his birth because the Communist authorities had prohibited the publication of his research papers and his participation in international congresses. After spending two years in a political refugee camp in Turkey, he emigrated to the United States.

Robert Muller of The Number Theory Publishing Company, PO Box 42561, Phoenix, Arizona 85080, USA, decided to publish a selection of his papers, commencing with *The Smarandache Function Journal*, Vol 1, No 1, December 1990. ISSN 1053-4792.

PCW readers may have met this function before, in Numbers Count -112- July 1992, where a very encouraging response was generated. This article [February 1993] is complete in itself so don't worry if you have filed the July issue! It may be thought that those readers who attempted the previous problem-set will have an unfair advantage. However, it must be realised that no Numbers Count problems are completely original so previous work within a given subject area is always a possibility and the prize is awarded using 'suitable subjective criteria' anyway, *so please have a go and submit your results*, however trivial they may seem to yourself.

Definition For all non-null integers, n, the Smarandache Function, S(n), is defined to be the smallest integer such that (S(n))! (The Factorial Function with argument S(n),) is divisible by n, e.g. S(18) = 6 because 6! is divisible by 18 but 1!....5! are not.

Problem (0) Design and implement an algorithm to generate and store/tabulate S(n) as a function of n upto a given n_{max}.

Hint It may be advantageous to consider the STANDARD FORM of n, *viz* n = $e p_1^{a_1} p_2^{a_2} p_3^{a_3} \ldots\ldots p_r^{a_r}$ where e=±1, and $p_1, p_2, p_3 \ldots p_r$ denote the distinct prime factors of n and a_1, a_2, $a_3 \ldots a_r$ are their respective multiplicities.

NOTE S(n) is an even function, by which is meant S(-n) = S(n).

Problem (i) Using either graphical or finite difference technique (i.e. the construction of difference tables etc) or indeed anything else that comes to mind, address the following questions:
(a) Is there a closed expression (formula) for S(n)?
(b) Is there a good asymptotic expression for S(n)? (By which is meant a formula, which although never (in general) exact, becomes a better and better approximation to S(n) as n becomes larger and larger.)

Problem (ii) For a specified non-null integer m, under what conditions does S(n) divide the difference n - m?

Problem (iii) Investigate the possible integer solutions, (x,y,z) of $S(x^n) + S(y^n) = S(z^n)$ for any n greater than or equal to 1, e.g. examine the solution (5,7,2048) when n = 3.

(It can be proved that an infinity of solutions exist for any such n-value.) Compare with Fermat's Theorem re. $x^n+y^n = z^n$.

Problem (iv) Investigate the possibility of finding two integers n and k such that the LOGARITHM of $S(n^k)$ to the BASE $S(k^n)$ is an integer.

Problem (v) Recall that 'Gamma' defined as the limit as n tends to infinity of (1 + 1/2 + 1/3 + 1/4....+1/n - log(n)) exists, is known as Euler's Constant and is approximately 0.577.

Investigate the possible existence of 'Samma' defined as the limit as n tends to infinity of (1 + 1/S(2) + 1/S(3)....+1/S(n) - log(S(n)).

Problem (vi) Find the number of PARTITIONS of n as the sum of S(m) for 2<m≤n. See *PCW* August 1989 and February 1990 for other problems involving PARTITIONS of n.

Attempts at some, or all, of the above Problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 May 1993. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, Numbers Count -113- *PCW* August 1992: 'A midsummer miscellany'

Five Unsolved Number problems... these did in fact provide fun for many readers of the Numbers Count column.

Among the most creditworthy efforts, mention must be made of Ed Hersom who concentrated on Problem I, rejecting II as suffering from 'The Curse of Dimensionality' and III & IV being 'rather artificial'. Ed used an RML Nimbus 80186/8087 running at 8kH in Forth and used the 'through the other end of the telescope' philosophy to produce products p and then to ask if 2p + 1 is prime? Many interesting results, details from EH or MM, together with a proposal to combine with BIADDS (see *PCW* June 1992) thus define x to be BIPRIME if x = y + z + 1, y and z must also be BIPRIMES....5,7,11,17,23,29,31.. Is this sequence infinite? Any thoughts on developing such BIPRIMES for a future Numbers Count column?

Gareth Suggett omitted Problems II, IV & V on grounds of lack of time; was able to relate I to a result in Hardy & Wright and in particular looked at the effect of removing repetitions from the sequence (x). For Problem II Gareth created a file of all powers upto 3×10^6, being limited by the memory of his BBC and searched it for various values of k, in particular k = 6, and k = 14 had no solutions within his program range. Does anyone know of any?

This month's outstanding winner has to be Mr RPC Forman of 1 Weston's Yard, Eton College, Windsor, Berkshire SL4 6DB. Roderick addressed Problems I & II using Mathematica and FrameMaker to produce material of a very high order of sophistication. For reasons of space only summary conclusions can be given here: details from MM or RPCF.

Problem II Based upon numerical evidence the 'intrinsically unsafe' conclusion is that there are only finitely many solutions to $x^p - y^q = k$ for any particular k greater than zero, further that there is a special set S of odd numbers (3,7,17,21,25,29,31,..) such that the solution set is null if k is in 2S.

Problem I It looks as if for any particular n greater than zero, there are infinitely many n-primes and that their distribution is related to the asymptotic density function, see Hardy & Wright *The Theory of Numbers*, Fourth Edition 1960, page 368.

An outstanding piece of work. Many congratulations!

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.