Beginners start here! A pot-pourri of problems, presented by Mike Mudge.

'For it is unworthy of excellent men to lose hours like slaves in the labour of calculation which could safely be relegated to anyone else* if machines were used.'

Gottfried Leibniz (1646-1716)—the 'German' Father of the Calculus.

There follows a structured collection of problems designed to enable 'anyone else'* to familiarise themselves with some of the subject areas of elementary empirical number theory.

It is hoped that a large number of *PCW* readers will be encouraged to submit their attempts at a selection of these problems for validation and comment. All submissions will be examined in depth and individual replies will be forthcoming.

Note: No special software or programming expertise is required, and (some) results can be expected within seconds rather than hours of correct computing.

Further, there is no prerequisite in the subject area of mathematics... so come on — give it a try. Every problem has some relevance in number theory, together with an element of the unknown, often suggested by the optional 'Extension', and so hopefully, experienced contributors will have something to investigate.

Problem 1: A NIVEN NUMBER, N, is a positive integer such that the sum of its digits, S(N), divides N exactly.

Trivially, 1,2,3,4,5,6,7,8,9 are NIVEN NUMBERS.

10 is, since 1+0 divides 10 exactly, so are 12, 18, 20, 21, 24,...

Write a program to obtain all NIVEN NUMBERS less than 1000.

Extension: Replace 1000 by 10^p where p is positive integer input.

Problem 2: Clearly $47^2 + 52^2 = 17^3$: find the other eight sets of numbers, all less than 50, which satisfy, $a^2 + b^2 = c^3$, i.e. pairs of numbers whose squares add up to a perfect cube.

Extension: Replace 50 by 10^p where p is a positive integer input, suggest how the replacement for nine sets above depends upon p.

Problem 3: 50 is the smallest integer which can be written as the sum of two squares in two different ways:

$50 = 1^2 + 7^2 = 5^2 + 5^2$

while 325 is the smallest integer which can be written as the sum of two squares in three different ways:

$325 = 1^2 + 18^2 = 6^2 + 17^2 = 10^2 + 15^2$

Write a program to express all n upto 10000 as the sum of the smallest possible* number of squares.

* Trivially $n = 1^2 + 1^2 + \ldots$n-terms.

Extension: Investigate directly the smallest number that can be expressed as the sum of q-squares in d-different ways, for q,d = 2,3,4,...

Problem 4: In 1848 de Polignac conjectured that every odd integer is the sum of a prime and a power of 2.

Thus $25 = 17 + 2^3$; find a* decomposition of this form, *viz* $(2n + 1) = p + 2^m$, for all odd numbers less than 1000.

Extensions: (a) Replace 1000 by 10^r, where r is positive integer input. (b)* Examine the possibility of multiple decompositions, note that $25 = 21 + 2^2$ is not acceptable since 21 = 7x3 is not prime.

Problem 5: The largest perfect square whose digits are strictly decreasing in order is $961 = 31^2$. Find the only other perfect square with this property.

Extension: Replace perfect square by perfect cube, fourth power etc, and then relax strictly decreasing to become non-increasing!

Problem 6: An AUTOMORPHIC NUMBER is an integer whose square ends with a repeat of itself. Thus 6^2=36, 25^2=625, 76^2=5776.

Write a program to identify all AUTOMORPHIC NUMBERS less than 10^p, where p is a positive input integer.

Extension: Consider TRIMORPHIC NUMBERS defined similarly in terms of their cubes, e.g. 875^3=669921875

Problem 7: A PALINDROMIC NUMBER reads the same in each direction. There are many (an infinity?) PALINDROMIC COMPOSITE NUMBERS which factorise into PALINDROMIC PRIMES, 1111 = 11 x 101, 1441 = 11 x 131, 3443 = 11 x 313.

Write a program to find other composite palindromic integers whose palindromic prime factors consist of digits greater than 3.

Extension: STROBOGRAMMATIC NUMBERS are numbers that read backwards after having been rotated through 180°, e.g. 69, 96, 1001...

Investigate Problem 7 above when these replace palindromes.

Many of the problems posed above were inspired by the publication *Invitation to Number Theory with Pascal* by Donald D Spencer, ISBN 0-89218-126-5, Camelot Publishing Company, FL 32075, USA, 1989. This book is highly recommended for beginners in empirical number theory. Programming ability in Pascal is not essential for its comprehension.

Attempts at some or all of the above problems and their extensions may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 June 1993. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded, by *PCW*, to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of coding, run times and a summary of results obtained, all in a form suitable for publication in *PCW*.

Additionally, comments expressing preference, or otherwise, for a collection of simply posed problems (as this month) compared to one or two more elaborate research areas would be appreciated.

Please note that all contributions will be critically evaluated and individual responses will be returned. However, the original submission can only be returned if a suitable stamped addressed envelope is provided.

Review Numbers Count -116- November 1992: A 'train' of thoughts from India

Paul Rayner is to be congratulated on finding all RARE NUMBERS upto 10^{17} in less than 350 hours using Turbo Pascal on a 20MHz Viglen III: 35 are palindromic and 42 are not. The largest is 67725910561765640; only two have an even number of digits, but all have a most significant digit, either 2, 4, 6, or 8.

However, the prizewinner this month is Adam Robinson of Hertfordshire, who investigated all aspects of the 'train' using an Acorn Archimedes running RiscOS 3.1, 'which is generally quite speedy, even when multitasking within the desktop environment.'

Adam calculated the Digit Product Termini of all primes upto 700000, his rare numbers investigation was limited to query (i) by the well-known integer arithmetic difficulties, Eporns were examined upto 230000. However, the major considerations in the prize award this month were style of programming, accompanying documentation and some pleasing theoretical results on Digit Product Termini. Any correspondence for Adam Robinson should be directed either to Mike Mudge or to the Production Editor, *PCW*.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.