A summer miscellany

Beginners continue from here! Another pot-pourri of problems posed by Mike Mudge.

The very considerable response generated by 'A pot-pourri of problems', *PCW* May 1993, has resulted in this second presentation of a structured collection of problems requiring a minimum of mathematical background yet providing an opportunity for new discoveries to be made by true beginners to empirical number theory.

The writer is indebted to the Pergamon Press for publishing a translation from the Polish (due to A Sharma) of 'A Selection of Problems in the Theory of Numbers' by Waclaw Sierpinski, *Popular Lectures in Mathematics*, 1964.

Problem 1 Is every number which leaves a remainder other than 4 or 5 when divided by 9 expressible as the sum of the cubes of three integers? In particular it is not known if the number 30 is expressible as the sum of three cubes... JCP Miller & MFC Woollett, *J London Math Soc* 30 (1955) pp101-110, 'Solutions of the diophantine equation $x^3 + y^3 + z^3 = k$'.

Problem 2 Can every natural number be put in the form $x^3 + y^3 + 2z^3$ where x, y & z are integers?

76 and 99 are the smallest numbers for which this has not been shown. Chao Ko, *J London Math Soc* 11 (1936) pp218-219, 'Decomposition into four cubes'.

Makowski A *Acta Arith* 5 (1959) pp121-123, 'Sur quelques problemes concernat les sommes de quatre cubes'.

Note K Moszynski & J Swianiewicz have found that $113=-133^3-46^3+2.107^3$.

Problem 3 Note that $x^3 - y^3 - z^3 - t^3 = 1$ has infinitely many solutions in natural numbers x, y, z & t because: $x^3 + y^3 + z^3 + t^3 = 1$ have infinitely many?
e.g. $4^3+4^3+6^3-7^3=1$,
$4^3+38^3+58^3-63^3=1$.
(Proved yes by JA Gabovitch in 1962.) How can these be sensibly tabulated?

ARITHMETIC PROGRESSIONS (APs) An AP is an ordered sequence of numbers each differing from its predecessor by the same fixed quantity, calld the common difference of the AP. e.g. 7,13,19,25 represents an AP with common difference 6.

Problem 4 Do there exist infinitely many APs formed of the squares of three difference primes? e.g. $7^2,13^2,17^2$ and $7^2,17^2,23^2$. It is interesting to observe that there cannot exist APs consisting of four different squares of natural numbers (Fermat) nor of three cubes nor of three fourth powers of natural numbers. *Colloq Math* 5 (1958) pp11-15, 'On the equation $x^3+y^3 = 2z^3$' by A Wakulicz.

Problem 5 If the numbers $1,2,3,....n^2$ are written in n — rows each of n — numbers, will each row contain at least one prime? This is known to be true for n upto 4500 (A Schinzel), but consider displaying it and examining the distributions of the primes. If the result is true then there is at least one prime between n^2 and $n^2 + n$ but is there at least one between n^2 and $(n+1)^2$?

Problem 6 A theorem of Dirichlet taken together with identity: $18k + 1 = (2k + 14)^3 + (3k + 30)^3 - (2k + 23)^3 - (3k + 26)^3$ ensures that there are an infinity of primes of the form $x^3 + y^3 + z^3 + t^3$ but what about the form $x^3 + y^3 + z^3$ where x,y,z & t are integers? GH Hardy & JE Littlewood conjectured the answer to be in the affirmative. *Acta Math* 44 (1923) pp1-70, 'Some problems of partitio numerorum III'. But could we easily obtain some empirical evidence to support this conjecture?

Now, in all the above problems where there are questions of the form 'Do there exist infinitely many...?' it is clear that no amount of computing can provide the answer. However, empirical evidence may reveal patterns or lead to provable alternative questions and so does, in my opinion, have an important role to play.

Attempts at the analysis of some, or all, of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 October 1993. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded, by *PCW*, to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of coding, run times and a summary of results obtained all in a form suitable for publication in *PCW*. Additionally, comments upon the subject matter together with any references to other related results would be of considerable interest.

Review of Numbers Count -119- The Logical Analysis of Chinese Checkers, *PCW* March 1993. Suggested by Michael Meierruth of Milan, Italy

Many readers recognised that the game referred to in this article was indeed better known, certainly within the United Kingdom, as Pegboard Solitaire; responses ranged from the very philosophical to the very mathematical. John Gilder of Cheadle, Cheshire typified the philosophers with the initial quotation, 'A man with a hammer is in danger of treating everything as if it was a nail.' He continues with the obvservation that to play on a well-crafted board is much more pleasurable than to indulge in any amount of blind computing. He concludes with the reference to an OUP paperback *The ins and outs of Peg Solitaire* by John Beasley and the advice to 'let your keyboard gather cobwebs and jump a few pegs.'

Gareth Suggett, one of the most regular respondents to Numbers Count over the past ten years, decided to add nothing to the comprehensive discussion in Berlekamp, Conway and Guy: *Winning Ways*, chapter 23.

However, the very clear prizewinner is E Tottenham of 194, Chaussee de la Hulpe, 1170 Bruxelles, Belgium. Five years of intermittent work using initially Turbo Pascal on an IBM PC/XT followed by Assembler on a PDP 11/73 and then Pascal on a VAXstation with runs of upto 64 hours making 40500000000 moves and finding 1230000 winning games (615000 with the peg in the centre) represents an enormous empirical study. All very well documented. Details from myself.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.