In the line of fire

Mike Mudge investigates the Omni-Riflemen problem, an exercise in simulation and logic.

The problem considered this month has an uncertain history; however, a recent telephone call from an experienced computer consultant, Phil Bowles of London, prompted its appearance in this column.

The problem is in two parts, the first involving a simple simulation using random numbers: I propose the resurrection of Quasi-Random Numbers whose origins with RD Richtmayer are in pure number theory (Uniform Distributions Modulo 1) as an alternative to the Lehmer Congruence Generators to be found in many software packages. The second part (after mass murder has taken place!) involves considerable logic and intuition if a final conclusion is to be drawn.

The Scenario A plane area of arbitrary size and shape. An unknown number, say N, riflemen, materialise at random points within this area, as if dropped by parachute.

At a given instant of time each rifleman shoots dead the fellow rifleman who is nearest to him. What is the expected (average) fraction of the initial N-riflemen who survive this mass murder?

Random Number Generation Many readers will have used random number generators in simulation programs and indeed for a variety of other purposes, but may have simply used a standard function provided as part of a software package. It is very likely that such generators are of the Lehmer (Additive, Multiplicative or Mixed) Congruence Type.

An alternative, yielding so-called Quasi- rather than Pseudo Random Numbers, involves taking any algebraic irrational number such as the square root of 2, and forming the fractional parts of its integer multiples. These are 'in some sense' random in the interval from 0 to 1. An independent sequence of such numbers is obtained from an independent algebraic irrational, say the cube root of 5. Such two independent sequences might, for example, provide the x & y co-ordinates of the riflemen. *Warning*: the generator that is described above can go dreadfully wrong, i.e. cease to produce Quasi-Random Numbers if problems of round-off are allowed to arise, thus it is essential to ensure that the digits being used for the simulation have indeed arisen from genuine digits of the algebraic irrational.

Having generated the positions of the N-riflemen, the algorithm for finding nearest neighbours needs careful thought else gross inefficiency will arise for large N. Finally, what is the expected fraction of survivors?

Extensions, which may or may not be meaningful:

1) Do the survivors form a random pattern?

2) What happens if a re-entrant area is permitted, and shooting cannot pass over the border?

3) Suppose the terrain is three dimensional and a line of sight requirement is introduced?

4) If the mass murder function is iterated, what is the probability of a given final result, and the expected number of iterations to attain it?

Now to something more precise — Stirling's Approximation to n!

While analysing the chess board problems posed in the July 1993 issue of *PCW*, in particular the N-Rooks problem, Stephen Poley of Ridderkerk, Holland, was using Stirling's Approximation to n! in the form (n/e)**n*sqrt(2*PI*n) when he recalled from some computational experimentation carried out several years ago that the accuracy was improved, at least for values of n upto a few hundred, by multiplication by (1 + 1/(12*n)). Stephen asks why this works and whether it is valid for all n?

Attempts at the Omni-Riflemen Problem, together with suggestions for Stephen Poley, may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 November 1993. It would be appreciated if such submissions contained a brief description of the hardware used, details of coding run times and a summary of the results obtained. Additionally, comments upon the subject matter together with any references to other publications dealing with this month's problems would be of considerable interest.

Review of Numbers Count -120-, *PCW* April 1993: A Quadratic Iteration and a PI Challenge. With suitable acknowledgements to Fred Salt of Bridgend and Eric Adler of London

Dealing first with the PI Challenge; a detailed derivation from Nigel Backhouse referring back to the ideas of Archimedes and citing PE Trier, 'Pi Revisited', published in the Bulletin of the IMA, vol 25 (1989), pp74-77, was typical of many received. David Broughton discussed matters of accuracy and referred readers to *The Art of Computer Programming, vol 2: Seminumerical Algorithms* by Donald Knuth, section 4.2.2. *Accuracy of FP Arithmetic,* Dr AN Brooks, relates Adler's program to the work of Vieta (1540-1603) and gives a proof by induction. Reg Bond suggests we view JM Borwein & PB Borwein *PI and the AGM*, John Wiley 1987, £64. Alan Cox expressed some concern at the magnitude of computing hardware referred to, and together with others suggested the use of Wallis's Formula for PI.

Meanwhile, Eric Adler has sent me a program to calculate PI from 3. Copies on request, SAE please.

The Salt iteration gave rise to many extended and very detailed responses: Reg Bond would like a complete theoretical solution to $x_{n+1}=x_n-x_n^2+u$... Paul Rayner must be congratulated on his extensive graphical output, on an Epson LQ500 printer from a 20MHz Viglen III programmed in Turbo Pascal. He and others saw the significance of The Feigenbaum number 4.6692.. as the ratio of (u(n)-u(n-1))/(u(n+1)-u(n)).

The first ever response from Bucharest, Roumania, used an Amiga 500 to demonstrate the bifurcation process, thanks to Muresan Dan. The very worthy prizewinner, however, is a new contributor, Martin C Hall, of Tree Cottage, Upper Basildon, Berks RG8 8NU, who ensured his success by posing the iterative scheme: $Y_{n+2} = C^*Y_{n+1} - D^*Y_n$ producing sine curves.

I hope many readers will write to Martin explaining what is going on. Surely as we all help one another with computing problems, a little mathematical help will also be forthcoming.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.