Repunits & Repetends

Some problems involving Repunits & Repetends, presented by Mike Mudge.

A base 10 REPUNIT, R_n, is a decimal number written as n 1's. A base 10 REPETEND is the shortest repeating string of digits in a repeating decimal number. The concepts of REPUNITS and REPETENDS are fascinatingly interwoven, the current research and recreation in this subject area being presented in an inspiring and unique manner by Professor Samuel Yates, *REPUNITS and REPETENDS*, Star Publishing Company, Florida 33435, USA; 1982. See also Numbers Count -15- *PCW* April 1984.

R_2, R_{23} & R_{317} are known to be the first three PRIME REPUNITS: the factorisation of REPUNITS is clearly a tedious and probably rather boring exercise in general. Restricting arithmetic to BASE-10- it is known that REPUNITS can never be perfect squares, but is it possible for them to be cubes or higher powers?

Problem 1 In an arbitrarily chosen number base is it possible for a REPDIGIT (a number written as n d's, where d denotes any permissible digit) to be a PERFECT SQUARE? See CW Trigg, *Journal of Recreational Mathematics*, 13:3, 1980-81, pp181-182.

SMITH NUMBERS A Smith number, defined by A Wilansky to be a composite (non-prime) number the sum of whose digits is equal to the sum of the digits of its prime factorisation (excluding 1) e.g. 4937775 = 3x5x5x65837, can be constructed as 3304 x R_n when R_n is prime. How many multipliers can replace 3304, and further, is the set of SMITH NUMBERS infinite?

See S Oltikar & K Wayland, 'Construction of Smith Numbers', *Mathematics Magazine*, vol 56.

Problem 2 Investigate the construction and distribution of SMITH NUMBERS and their relationship with PRIME REPUNITS.

COFACTORS OF REPUNITS These are of two distinct types: (i) the PRIMITIVE COFACTOR of a REPUNIT is that factor of a repunit whose divisors divide no smaller repunit; (ii) the ALGEBRAIC COFACTOR of a REPUNIT is that factor of a repunit each of whose prime divisors divides at least one smaller repunit. e.g. If A_n denotes the Algebraic and Pn denotes the Primitive cofactor of the Repunit $R_n = (10^n - 1)/9$ we see that: *(see below)*.

n	1	2	3	4	5	6	10	18	22
A_n	1	1	1	P_2	1	P_2P_3	P_2P_5	$3P_2P_6P_9$	$11P_2P_{11}$

Now consider the case when n = 6x say, for integer x. $R_n = R_{n/2}(10^{n/6} + 1)(10^{n/3} - 10^{n/6} + 1)$ denote the rightmost factor by B_n: this is easily seen to be a concatenation of n/6 9's followed by n/6 0's plus 1. Thus: when x = 1 B_6 = 91, when x = 2 B_{12} = 9901 etc.

Problem 3 When is B_n necessarily composite, and under what conditions can B_n be prime? When is B_n equal to the primitive cofactor of R_n?

In 1927, the primality of B_n for n less than or equal to 10 was verified. DH Lehmer found that B_n is prime when x = 2,4,6 & 8 but composite when x is 10 or odd in that range. (It is not thought to be the case that B_{6912} and B_{8748} have been factorised.)

Something Else... Found in The Pengiun Dictionary of Curious and Interesting Numbers by David Wells. Reprint 1987, £4.99, and thought to be incredible by MM and others:

Consider the interval (0,1), choose numbers a,b,c,d etc in this interval so that: a&b are in different halves of the interval, a,b&c are in different thirds of the interval, a,b,c&d are in different quarters of the interval and so on. Not more than 17 such numbers can be so chosen.

Problem 4 Write a computer program to select points at random in the interval (0,1) and test that the above criteria are satisfied. Experiment extensively to see how many points are generated before the criteria fail. Does this number ever become greater than 17? If not, why not?

Attempts at the analysis of some, or all, of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel: (0994) 231121, to arrive by 1 December 1993.

Review of Numbers Count -121- *PCW* May 1993: Beginners start here!

This proved to be the most successful column in the ten years of Numbers Count, measured both in terms of quantity and quality of the responses received.

A complete set of attempts from David Knight of the University of Glamorgan was more than matched by Henry Ibstedt who searched also for strobogrammatic numbers with strobogrammatic factors upto 10^6, finding only 1111, and composite palindromes with palindromic prime factors having digits greater than 3 upto 2300000032 (even numbers of digits) and 230000032 (odd number of digits) finding only 343. Many readers understandably concentrated on one or two problems: Alan Stokell evaluated Niven's to 10^7, David Foster-Morgan found a 45-digit solution to Problem 6 part (i), Peter Winstanley used Sharp APL/PC to produce very economical code on a 286 clone on both problem (i) and a generalisation of problem (ii) $x^n + y^n = z^p$. Nigel Hodges gave a most interesting theoretical analysis of problem (iii); while another complete set of attempts in Aztec C on a Commodore Amiga arrived from Muresan Dan resident in Bucharest. Damian Chapman did some valuable work in Borland Turbo Pascal v6.0 using a PC compatible computer with an Intel 286 chip running at 16MHz, but failed to break the barrier of Maxint = 32767.

Many thanks to everyone — in particular the very worthy prizewinner, Anthony Isaacs, of 46 Bramber Road, North Finchley, London N12 9NE, who used Salford University FTN77/486 version 2.51 on an Elonex 386B 25MHz with 80387 co-processor and 4Mb RAM. Details from MM or AI.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.