# Chained up

## Cunningham Chains (Sophie Germain Primes) and Prime-Valued Polynomials, presented by Mike Mudge.

The topic of Cunningham Chains has been suggested by a regular reader, Tiger Redman, of March, Cambridgeshire.

David Wells' book, *Curious and Interesting Numbers*, Penguin 1987, states: 'A Cunningham Chain of Prime Numbers is a sequence in which each prime is 1 more than twice the previous member. DN Lehmer determined that there were only 3 such chains of 7 primes each with the first member less than $10^7$.'

The smallest such chain, due to RK Guy, is 1122659,-,-,-,-,-,71850239. Now these numbers are, in fact, Sophie Germain Primes: i.e. p is an SG-Prime if and only if (iff) 2p + 1 is also prime. The recent publication, *The Little Book of Big Primes*, Paulo Ribenboim, Springer-Verlag, 1991, ISBN 0-3-540-97508-X, £20.50, informs readers that the largest SG-Prime is 39051 x $2^{6001}$-1 (W Keller 1986); other large SG-Primes include 296385 x $2^{4251}$ - 1 & 153375 x $2^{4203}$ - 1 (J Brown et al.) Now, the length of a Cunningham Chain (consisting, by definition, of SG-Primes) is defined in the obvious way as , k, the number of primes in the chain.

**Problem NOT TO BE SOLVED BY COMPUTER...** Is there a Cunningham Chain of length k for every positive integer k?

**Related Problem** If the above chains are named Cunningham Chains of the FIRST KIND, while those of the SECOND KIND are defined requiring each prime to be 1 less than twice the previous member, is there a Cunningham Chain of either kind of length k for every positive integer k?

The largest known chain of SG-Primes has length 12 and its initial prime is 554688278429, while the latest known Cunningham Chain of primes of the second kind has length 13 and smallest prime 758083947856951. These results are due to G Löh (1989) and quoted by Paulo Ribenboim.

**Problem 1** Design and implement an algorithm to construct Cunningham Chains of both the first and second kinds. Consider also possible generalisations, i.e. instead of 2 x p ± 1, what about n x p ± m? What constitutes a set of interesting (m,n) pairs?

**Prime-Valued Polynomials**

Readers whose familiarity with *PCW*'s Numbers Count extends back to October 1988 may recall the earlier visit to this topic. However, a number of new results and alternative approaches to this investigation have come to light: see in particular Paulo Ribenboim, ref above.

This study must commence with $F(X)=X^2+X+41$, Euler (1772) which is prime-valued for X=0(1)39 taking values 41,...1601. F(40) = 1681. F(40) = 1681 = $41^2$.

This is still the best (i.e. longest consecutive string of prime values) polynomial of the form $X^2+X+q$. However, if the quadratic is generalised to $aX^2+bX+c$ the 'best' (a,b,c) triples cited by Ribenboim are (36,-810,2753); (103,-3945,34381) & (47,-1701,10181) having strings of prime values of lengths 45, 43 & 43 respectively. The first two are due to R Ruby and the last to G Fung.

**Problem 2** Investigate the lengths of prime values, for consecutive positive integer X, of $aX^2+bX+c$ for ranges of (a,b,c) verify, and hopefully extend, the above results. Can longer strings be obtained if the starting value of X is different from 0?

**Polynomials of Degree 1, i.e. f(X)=dX+q** There are at most q successive prime values; these are the moduli (absolute values) of f(0),..f(q-1): e.g. (q,d)=(3,2) yields 3,5,7. (q,d)=(5,6) yields 5,11,17,23,29 while (q,d)=(7,150) yields 7,157,307,457,607,757,907.

**Problem 3** Investigate empirically the conjecture 'so difficult that I believe no-one will prove it, nor find a counter-example in the near future...' (Paulo Ribenboim)... that for every prime q there exists an integer d greater than or equal to 1 such that q, d+q, 2d+q,....(q-1)d+q are primes.

In October 1988 readers of this column were introduced to the density function: V(f(x),N) defined as the number of integer (non-negative) x values less than or equal to N for which f(x) is prime, or equal to unity.

Some problems were posed involving quadratic f(x). However, in 1989 P Goetgheluck suggested a 'race' for cubic polynomials, like the Indianapolis 500 (!), to maximise V(C(x),500) where $C(x) = ax^3+bx^2+cx+d$, i.e. the general cubic polynomial.

The probable winner is $2x^3-489x^2+39847x-1084553$ in the 'heats' for which a is restricted to the values 1 or 2, with a V of 267.

**Problem 4** Run the cubic polynomials-500 race both for heats having a = 1 or 2 and for other values. Provide a summary of probable winners, hopefully including the above cubic or its associated victor.

Attempts at the analysis of some, or all, of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 January 1994. Any communication received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. Please note that material can only be returned if a suitable stamped addressed envelope is provided.

**Review of Numbers Count -122-, *PCW* June 1993: A-&U-Numbers**

Gareth Suggett and others corrected the length of the A-sequence from 189 to 184, the largest term being the 113th at 33289162099526. Paul Rayner explored the 'seed' of 276 as far as the 585th term which has 56 digits; in addition, his extensive analysis using UBASIC86 on a 20MHz 386 considered every 'seed' upto 5000, halting when either a term exceeded $10^{20}$ or the number of factors exceeded 1200 or the sequence became periodic or reached a static end value. Results may be obtained from Paul, enquiries via Mike M.

The prizewinner this month is a newcomer to the *PCW* readership (the magazine being given to him by his boss!): Peter Copeland of 'Rock House', 19 St George's Hill, Easton-in-Gordano, Bristol BS20 0PS. Paul has been studying A-sequences since he acquired a Texas Instruments TI57 in 1976; he has carefully classified the observed behaviour and generated extensive results on an 80486DX running at 66MHz. Unfortunately space does not permit their display; some details on request. Well done Paul, and very well done Peter!

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.