Three of a kind

Three unsolved problems from Geometry, including car driving and furniture moving!, presented by Mike Mudge.

The inspiration for this month's problems has been provided by the study of *Unsolved Problems in Geometry*, by HT Croft, KJ Falconer & RJ Guy (Springer-Verlag 1991).

POLYOMINOES An n-omino or a polyomino of order n is the union of n squares from a square grid joined edge to edge to form a simply connected block (i.e. a block with no holes in). The term was first used by SW Golomb in 1953 and his extensive research produced the book *POLYOMINOES* (Allen and Unwin, London 1966) followed by a number of other papers.

Problem 1 How many n-ominoes are there for each n? Denote this number by N(n) and attempt to confirm and extend the table in Fig 1. DH Redelmeier, 'Counting polyominoes: yet another attack'; *Discrete Math*, vol 36 1981 pp191-203, reached n = 24.

Fig 1

n	1	2	3	4	5	6	7	8	9	10	11
N(n)	1	1	2	5	12	35	107	369	1285	4655	17073

Fig 2

n	3	4	5	6	7	8	9	10	11	12	13	14
t(n) Greater than or equal to:	3	3	4	3	3	4	6	5	6	6	6	7

What type of formula might be used to provide a good approximation to N(n) for large n? i.e. an asymptotic approximation.

Extensions that may be considered include POLYIAMONDS formed by triangles taken from the regular triangular grid, POLYHEXES obtained using the regular hexagonal grid, and naturally in three dimensions POLYOMINOIDS obtained by glueing a collection of unit cubes face to face in some arrangement.

Some Dynamics in Two Dimensions

Problem 2 (A)

Moving Furniture Around

Leo Moser Problem 66-11, *SIAM*, Rev 8 (1988) p381 asked for the region of largest area that can be moved around a right-angled corner in a corridor of unit width. Note: this is to be regarded as a two dimensional problem. A structure called the Shepard Piano has an area of about 2.2074 and is a good first approximation; it has been modified to 2.21528, 2.21563, 2.21503 and 2.2115649 by C Francis, RK Guy and others. What kind of shape can you devise? What is the area of your shape?

Problem 2 (B) Reversing a generalised car into a T-Junction

What is the largest region that can reverse into a T-Junction with all roads of unit width? No clues given here: submit the shape and area of the largest region you can find!

Both the above problems can be modified by requiring that the regions be convex, by which we mean that any chord (straight line joining two points on the perimeter of the shape) lies totally within the shape.

There are higher dimensional analogues of both problems, but these are virtually intractable.

Ordinary or GALLAI Lines

Given n points in a plane, not all on one line, any line which passes through (or contains) exactly two of these n points is called an ordinary or GALLAI line. The number of such lines determined by the given points is denoted by t(n).

Problem 3 What is the minimum of t over all possible configurations of n points?

The minimum is known to be at least one; this follows from a classic result known as Sylvester's Problem. The values shown in Fig 2 are also known. In particular, interest centres upon a possible formula for the minimum value of t for odd n greater than or equal to 15.

Readers who find these geometrical problems of interest may wish to consider a supplementary problem:

DISC COVERING The problem of completely covering a circular region by placing over it, one at a time, five smaller equal circular disks was familiar to frequenters of English fairs a century ago. It can be done if the radius of the smaller disks exceeds 0.609383.. of that of the circular region. EH Neville, 'Solutions of numerical functional equations', Proc. London Math Soc (2) vol 14 (1915) pp 308-326.

What is the minimum radius for coverings by other numbers of equal discs? The cases of three, four and seven are easy (RK Guy *et al*); solutions for five and six are known and there are conjectured extremals for other numbers up to twenty. CT Zahn.

Can these results be approximated to by the use of some kind of interactive graphics on a PC?

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 March 1994. Comments upon this month's choice of problem area together with any references to other related results, published or known to the writer, would be of interest.

Review of Numbers Count -124- *PCW* August 1993: Full Houses of Primes and The Euler Totient Function

Some remarkable results were generated in response to these problems. Paul Rayner used UBasic on a Vig III (20MHz 386 co-processor) and in about 24 hours listed the 8096 full houses below 200000000. Using a least squares curve-fitting algorithm for results above 200000 he found that the number of full houses approximates within about 2% to:

$$\frac{P_n^{0.72550215}}{133.01757}$$

Would any *PCW* reader like to modify these Rayner Parameters, and if so, how?

After careful consideration, this month's worthy prizewinner is Tiger Redman of 26 Heathcote Close, March, Cambs PE15 9BD, who worked on his BBC B with considerable ingenuity. Regarding the average height of The Euler Totient Function it must be observed that Paul Rayner suggests 1.2202 ± 0.0001 whereas Jim Duncan prefers 'probably 1.23(3)': a good agreement for such a tedious investigation.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.