Something simple

Some simple applications of logic and programming skills in empirical number theory, presented by Mike Mudge.

The selection of topics this month is intended to encourage *all* readers to respond! There is no requirement to possess, or develop, such software as general precision arithmetic (often essential to make real progress in empirical number theory). There is felt to be great scope for application of logical thought and efficiency of programming technique, both of which are fundamental to sensible application of any computer hardware, in whatever field is found attractive to the user.

The author (MM) has received considerable inspiration from the recent publication *Primes and Programming, An Introduction to Number Theory with Computing*; Peter Giblin, Cambridge, 1993: ISBN 0-521-40988-8, £12.95 — highly recommended to all Numbers Count readers.

Fibonacci Numbers (FN) with an application
The FN are a sequence of positive integers defined by:

$f_1=1$, $f_2=1$, $f_n = f_{n-1} + f_{n-2}$ if n greater than 2. These have been introduced to readers before, see *PCW* May/September 1983. However, apart from the definition, there is no repetition of material.

Problem 1 A remarkable theorem, JP Jones, 'Diophantine representation of the FN', *Fibonacci Quarterly* vol 13, 1975, pp84-88, states that the FN are precisely the positive values of the fifth degree polynomial:

$F(a,b) = 2ab^4 + a^2b^3 - 2a^3b^2 - b^5 - a^4b + 2b$, where the a & b are non-negative integers.

Investigate this result numerically with the aim of predicting the a & b values needed to produce a given f_N.

Problem II David Wells informs us, in his famous *Penguin Dictionary of Curious and Interesting Numbers*, that 510510 is equal to the product of the first seven prime numbers: 2x3x5x7x11x13x17 and also equal to the product of four consecutive FN *viz* 13x21x34x55 — Monte Zerger, *Journal of Recreational Mathematics*, vol 12. Investigate the existence of other integers which are expressible both as a product of (consecutive) primes and of (consecutive) FN.

Problem III Design and implement an algorithm to express any positive integer (within the acceptable integer length of the system being used) as the sum of distinct and non-consecutive FN e.g. 54 = 2+5+13+34.

One version of the game of NIM
In this game, for two players, there exists a pile of n nails. The first player must take any number m_1, less than n of these; the second player takes any number m_2, less than or equal to $2m_1$; and so the game proceeds with players alternately taking at least one nail and not more than twice the number taken by their opponent in the previous turn. The last player to take nails from the pile is declared the winner.

Problem IV Design and implement a program to play this verson of NIM (HINT: could reasonably be called Fibonacci Nim!) as computer versus 'player'. As 'player', what are the chances of winning if (i) starting first, (ii) starting second?

Now to something else!
When is a fraction equal to an integer?
This is clearly too vague a question to be meaningful in general, but suppose the numerator and denominator are to be constructed from the digits (1,2...9) used once each only.

It is possible to construct fractions with five digit numerators and four digit denominators; thus 13458/6729 = 2, 14367/2589 is approximately 5.549. The first case is to be called a TYPE I(nteger) fraction.

Problem V There are 187 TYPE I fractions. Construct a listing of these.

How does the percentage of TYPE I fractions change:
(i) with the introduction of the zero?
(ii) when the relative numbers of digits in the numerator and the denominator are changed? (Trivially, if the former is less than the latter, the percentage is zero!)
(iii) when the number base is different from ten?

These three questions can, of course, be combined to generate an infinite set of exercises, but is there an underlying theory enabling the percentage of TYPE I fractions to be predicted in any particular case?

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 April 1994. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded, by *PCW*, to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief descripton of the hardware used, details of coding, run times and a summary of the results obtained. Additionally, comments upon this month's choice of problem areas together with any references to other related results, published or known to the contributor, would be of considerable interest.

Please note that material can only be returned if a suitable stamped addressed envelope is included.

Review of Numbers Count -125-, *PCW* September 1993:
'A summer miscellany', subtitled 'Beginners continue from here!'
A disappointing number of 'beginners' from May 1993 did in fact take up the offer to 'continue from here'; why? Replies on a postcard, please. Paul Rayner concentrated his efforts on Schinzel's Problem, number 5; first noting that a necessary but not sufficient condition is that the maximum separation which exists between adjacent primes below n^2 must be equal or greater than n.

Using Ubasic Nxtprm function on a 20MHz 386 the maximum separations for n^2 upto 1700000000 were calculated in about 120 hours, and subsequently analysed and displayed graphically. This was the basis of an investigation into LONELY PRIMES, defined as having the largest possible separation from both the greater and lesser adjacent primes.

However, this month's winner is David Broughton of 17 Golden Ridge, Freshwater, Isle of Wight PO40 9LE, who 'liked the challenge of this problem' (Problem 1) and was 'pleased to design an algorithm that did not require the cube root function and was still very efficient.' (Multiplication was also disposed of!) The question is still open but David has much empirical data — details on request.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.