In the interval

Mike Mudge sets his sights on Practical Arithmetic this month, with an introduction to Interval Analysis.

Readers are reminded of the mnemonic: GIGO translating as 'Garbage in Garbage out' and asked the question 'How is Garbage recognised?' Consider the following simple calculations:
3.48 + 2.24 = 5.72; 3.48 - 2.24 = 1.24; 3.48 x 2.24 = 7.7952; 3.48 / 2.24 = approximately 1.5536. SQRT(3.48) = approximately 1.8655.

The first three results are arithmetically exact, while the fourth can be obtained to any required number of decimal places; or indeed expressed exactly as $1.55\dot{3}57142\dot{8}$ or $1.553\overline{571428}$ using alternative notations to denote a recurring (or periodic) non-terminating decimal.

The fourth result, in contrast, is an irrational number. Its decimal expansion is both non-recurring and non-periodic and hence a precise answer as a decimal is not possible.

Now we suppose that the data, 3.48 and 2.24, are the result of experimental measurement or indeed some other form of estimation and hence have an inherent possible error. To be specific then, 3.48 represents a number which is equally likely to lie anywhere between 3.485 and 3.475. This shall be denoted by a new variable type called an INTERVAL and written (3.475, 3.48, 3.485).

It is to be noted that, depending upon the source of the data, the interval may not be symmetrical about the characteristic value. Thus (25, 35, 35) might denote the number of students expected to be present in a class with 35 enrolments; while particular measurements such as the estimate of the height of a vehicle might be intentionally always an over-estimate so that the lower-bound (LEFTMOST ENTRY) equalled the characteristic value.

The idea is to construct an efficient series of algorithms to handle these INTERVAL type variables: to input and output them, add, subtract, multiply and divide them and hence to carry out (efficiently) the whole spectrum of arithmetic functions. The process of taking the square root being the most obvious member of these latter operations?

PROJECT To design and implement a sequence of INTERVAL ARITHMETIC ROUTINES, culminating in the ability to solve a quadratic equation $Ax^2 + Bx + C = 0$, where A, B & C are REAL INTERVALS.

Beware... The width of the data intervals may be such that the roots can change type: i.e. from REAL to COMPLEX, in which case either the interval routines must be adapted to cope or an output message advises the user of the problem. (Hopefully suggesting a way in which the data intervals may be narrowed to avoid the occurence of such (undesirable or physically irrelevant) roots.) Recall the formula for finding the roots of: **$ax^2 + bx + c = 0$ is x = (-b ± Sqrt(b^2 - 4ac))/(2a).**

NEWS FLASHES

A/ A new Mersenne Prime* has been recently discovered. I acknowledge Nigel Backhouse of Liverpool as my source of this information, $2^{859433}-1$ having 258716 digits when expressed radix ten.

**For a discussion of Mersenne Primes see for example The Little Book of Big Primes by Paulo Ribenboim, ISBN 0-387-97508, Springer Verlag, 1991, page 65 et sequo. This was, I believe, done using a Cray mainframe... but how?*

B/ A number of respondents to the Numbers Count column have been successful in getting their results published in *The Smarandache Function Journal*, Vol 2-3, No 1, December 1993, ISSN 1053-4792. This journal originates jointly in the Department of Mathematics, University of Craiova, Romania**, and with Dr R Muller, Number Theory Publishing Co, PO Box 10163, Glendale, AZ 85318-0163, USA.

***Contact names Dr C Dumitrescu and Dr V Seleacu.*

Congratulations to all concerned. It is hoped these NEWS FLASHES will encourage readers to respond, even if they feel that their contribution is either well known or trivial.

Attempts at the above project may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 May 1994. Additionally, users' experiences with the generation of GARBAGE... in the sense of digits which are not justifiable from the implied accuracy of the given data, would be welcome.

EXAMPLE A rectangle measured as 2 by 3 appears to have an area of 6 but if the intervals are (1.5,2,2.5) and (2.5,3,3.5) the area interval is (3.75,6,8.75)! — a very different answer.

Review of Numbers Count -126- *PCW* October 1993: Incorporating the Omni-Riflemen together with Stirling's Approximation to N

This package produced a bumper response. Stephen Poley's requests regarding Stirling's Formula were also very productive. Particularly worthy of mention are Nigel Backhouse's efforts using MAPLE V on a 50MHz 8Mb RAM Opus 486 PC, to compute the first 30 terms in the asymptotic expansion in 4 seconds (the coefficient of n^{-29} having 75 digits in the numerator and 69 in the denominator), and Paul Rayner who developed an exponential correction which he later located in Alan Miller's book *Borland Pascal Programs for Scientists and Engineers*, p282.

Interpretations of the OMNI-RIFLEMEN and hence the choice of a suitable modelling technique proved to be quite controversial. Gareth Suggett proceeded cautiously with 1,2,3,4 riflemen only, the latter case yielding a ratio like 55:57:88 for 0,1&2 survivors. David Broughton reasoned that a one-dimensional simulation was adequate to predict in any number of dimensions. Alan Cox tentatively suggested induction over N to produce 'The Simple Function' E(N) giving the expected number of survivors from a population of N. Paul Rayner suggested about 28.5% survival until the grid is 5% populated then rising to about 31.6% at 50% population. Henry Ibstedt and John Scholes were among other very worthy responses.

However, the prizewinner is Michael Behrend, of 43A Fen Road, Milton, Cambridge CB4 6AD, using JPI Modula-2 plus Assembler on an Akhter PC clone containing an 8MHz 8088. Michael's limit being 28.5%, but not yet shown to be region independent. Details on request.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.