Ding dong!

A timely problem for holidaymakers, involving faulty station clocks, packaged and presented by Mike Mudge.

TYPE I. 'Where there's a will there's a way' or how can the correct time be determined from a faulty digital display?

This family of problems was suggested by Peter Skuse of Surrey whose family motto is used as an introduction. Consider the normal seven-segment display of a digit on a digital clock (Fig 1a), denoting the digit 8, all other digits being subsets of this display.

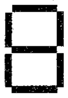
Fig 1a

If three segments are known to be faulty, in the sense that they show 'on' when 'off' and 'off' when 'on', what are the possible times which may be represented by a given display? Further, if it is known that one, two or indeed three segments are faulty in this sense, what is the minimum time for which the display must be observed in order to uniquely identify the faulty segments?

These problems can have a large variety of starting positions and may be associated with various types of display, e.g. H:M:S, H:M or M:S. Peter also hints at generalisations to matrix displays such as those on videos or sports or calculator displays with a digital component (Fig 1b).

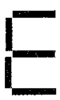
Fig 1b

However, his specific problem involves the station clock display as shown in Fig 1c with the malfunction in three elements. If there is no way of determining anything about the real time, how many minutes must the display be observed to determine which elements are faulty?

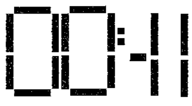
Fig 1c

TYPE 2. Related Number Bases or 'How to use the DONG!'
If an integer v is represented by a sequence of n digits e.g. abcde and their places in v by p (p = 1,2,3,..n) each digit is supposed to be multiplied by g**(n-p), g being an integer. Without any definition of number base g, number v is meaningless. Even this definition could be a problem for g itself requires a base!

Therefore we decide that n, p and g are numbers of the system for which counting means 0,1,2,3,4,5,6,... As v may represent different integers, one for each value of g, various interesting exercises are possible.

This introductory material and the abbreviation (RNB) for Related Number Bases (see below) were submitted by Fred Nooitgedacht, a retired professor from the University of Amsterdam, now living in Apeldoorn in The Netherlands.

Fred had originally developed this idea with regard to two provinces or a certain country using the same currency, the DONG. However, it is supposed that their official number bases are different.

Which number bases could occur if a particular article costing 123 DONG in one province cost 321 DONG in the other?

The generalisation involves alternative value pairs (v_1,v_2); each in the common unit, the DONG, but making equivalent purchases in the two provinces having number bases (b_1,b_2) (Fig 2).

Other rules for changing v_1 into v_2 apart from digit reversal can be considered, n may also be varied!

Even negative values of g could be considered, could g and -g be related number bases? If so, how would v_1 and v_2 be specified?

Attempts at both TYPE 1 and TYPE II investigations are encouraged. It is suggested that there is more scope for both logic and programming skill here than may at first appear to be the case.

Any material produced may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 August 1994.

Readers' comments upon the general or specific nature of this month's column, together with references to any related recent work, would be most valuable. This material, together with the overall size and quality of the response, is most essential in attempting to produce the 'Numbers Count' column which readers want.

Fig 2

v_1	123	234	147	369
v_2	321	432	741	963
b_1	17	44	19	17
b_2	29	62	49	29
Decimal Value	902	7878	2604	2706

Review of Numbers Count -129-: Root Filling, *PCW* January 1994
A significant contribution was received from Gareth Suggett who sought to examine the successive 'record' values for F(n) as n increased. A 'brute force' approach led from 12, 1.333 via 1260, 2.46667 to 166320, 3.29437. A speeding-up process was then invoked from the observation that consecutive groups are divisible by consecutive PRIMORIALS; this led (without rigorous proof) to a sequence of results limited by available time to:
4658179125600, 4.84225.

The very worthy prizewinner is however the Revd. Colin Alsbury, Industrial Chaplain, South Cheshire Industrial Mission, Crewe Green Vicarage, Narrow Lane, Crewe Green, Crewe, Cheshire CW1 1UN, using GW-Basic on an Amstrad PPC640 with a bit of help from Lotus 1-2-3! Details from MM or Revd. C.A.

Post Script on 'Something Else...': Numbers Count, November 1993

A recent communication from Michael Behrend of Cambridge asks who first proved the result for 17 points as stated, but provides another related result. Consider the two possible conventions about abutting intervals. If x<y<z, shall y be allowed to belong to both intervals (x,y) and (y,z)? If NO (the 'strict' convention) then the result as quoted is true. If YES (the 'lax' convention) then it is false and the required correction is to replace 17 by 23.

Michael asks if this is a new result and provides interesting run times using Zortech Demo C run on an Akhter PC clone with an 8088 running at 8MHz. The strict convention took 3m 15s with 19632 maximal configurations, while the lax convention took 39h 50m with 13314792 maximal configurations.

I suggest Michael should publish his results. What do other readers think?

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.