The final frontier?

How to get 1.136 litres into a 0.568 litre pot. With thanks to Arthur C Clarke, presented here by Mike Mudge.

This problem is a direct result of a recent communication from the eminent scientist and science fiction writer, Arthur C(harles) Clarke of Colombo 7, Sri Lanka. Out of deference to his reputation, the original wording is retained: 'TEXT COMPRESSION — THE FINAL FRONTIER?'

These comments have been triggered by a spoof item** in DATAPHILE, downloaded from INTERNET'S aus.jokes, describing how text of arbitrary length might be compressed into a single bit! It brought to my mind Frederik Pohl's famous story, 'The Gold at the Starbow's End' (circa 1970), which suggested that Goedel Numbers might be used to encode huge amounts of text in an extremely compact form — though one almost impossible to decrypt. (I once presented this argument at a Nobel Symposium in Sweden, and had the privilege of being shot down by Hannes Alfven!)

I would like to ask this question, which I have never seen answered:

Two digits (00 - 99) are more than sufficient to specify all the letters, numbers, cases, punctuation marks etc in ordinary use. The average word is about five letters long, so a thousand words of text can be encoded in a 10,000 digit number. If it consisted entirely of zeroes, this could be written very compactly as 10 e 10000 or 10 e 10 e 4. However, to convey useful information, every digit must have some arbitrary value between 0 and 9.

Problem: is it always possible to find strings to specify any desired long number, accurate down to the final digit, which are much shorter than the number itself? For example:

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11 e 9 e 5 + 9 e 6 e 4 - 5 e 3
e 2 + 2 e 7 e 2 - 10 e 7 + 471
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exactly describes an enormous number, thousands of digits long. Or can this be done only for so (relatively) few cases, out of the huge spectrum of possible numbers, that it's of no practical importance?

Over to the image compression algorithmatists++...

***Copies of the aus.joke article available from Mike Mudge, SAE please.*
++Otherwise known as 'Numbers Count' readers!

Repunits Again... A news update and some Repunit Riddles
Recall that R_n denotes the positive integer, base ten, represented by a string of n-ones. Nigel Backhouse of Helsby, Cheshire, has filled a gap in the published tables of REPUNIT factorisation using the ECM-Program (Elliptic Curve Method due to Lenstra, Jr, H.W., see Math. Inst., Univ.Amsterdam, Report 86-19, 18 pages; also Report 86-18, 19 pages; both 1986) supplied with UBASIC. The calculation, which took 360 hours on a 33MHz 486 PC, revealed that:

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R71 =
241573142393627673576957439049
x
45994811347886846310221728895223034301839
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A 30-digit prime times a 41-digit prime.

Now to R_{101}, R_{111}, R_{117}, R_{129}... and there are more!

However, REPUNITS have many RIDDLES associated with them which may not require any computing power for their solution. This sample is taken from The Journal of Recreational Mathematics and has appeared, in whole or in part, in Mathematics Magazine and also in that fascinating publication, Repunits and Repetends, by Samuel Yates, Star Publishing Company, Florida 33435, USA (1982).

RIDDLE 1

r_1. Find a pair of repunits whose product is a 100-digit palindrome.

r_2. What is the smallest repunit that is a multiple of the square of 183263? (Why did we choose this number...)

r_3. Solve the ALPHAMETIC: REPUNITS+REPUNITS+REPUNITS+ REPUNITS+REPUNITS+REPUNITS+ REPUNITS+REPUNITS+REPUNITS equals a repunit.

r_4. Solve the ALPHAMETIC: (EVE)/(DID) =.TALK where the right-hand side represents a recurring decimal.

r_5. If S = 1/9R1 + 1/9R2 + 1/9R3.....1/9Rn = ... evaluated as a decimal, what digit is to be found in the 37th decimal place?

r_6. Show that the 61 numbers beginning with 60! and ending with 60! + 60 are all composite, i.e. non-prime. Find an equally long string of consecutive composite numbers that are much smaller.

Solutions to $r_1..r_6$ obtained by brute force computing should, ideally, lead to a theoretical, simple derivation of the same results.

Responses to Arthur C Clarke's Problem, together with attempts at some, or all, of the REPUNIT RIDDLES (and indeed additional REPUNIT FACTORISATIONS) may be sent to: Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 September 1994. Communications received will be judged, using suitable subjective criteria, and a prize will be awarded, by *PCW*, to the 'best' contribution arriving by the closing date. Readers' comments upon the general or specific nature of this month's column would be most valuable.

Review of 'Numbers Count' -130- 'Some Applied Geometry in an Ideal World, *PCW* February 1994
Surprisingly, the POLYOMINOES PROBLEM produced a minimal response. It is proposed to re-open the study of these items and award a subsidiary prize (from Mike Mudge) for any 'worthwhile' submission. The topic has been used as a source of project-work at a variety of academic levels, so is clearly not as difficult as some readers perceive.

Shepard's Piano led David Broughton to acknowledge 'an interesting and fascinating problem which has given rise to quite a bit of interesting mathematical discovery.' On his IBM PC compatible with an 80386DX/33MHz CPU without a maths co-processor, running DESQview under PCDOS 6.1 and using 4DOS version 4 as the command line interpreter, he urges caution relating to any results in excess of 2.218 'although the ultimate figure may well approach 2.219.'

The very worthy prizewinner however is Fred Nooitgedacht, Pallietergaarde 219, 7329 HC Apeldoorn, The Netherlands, who received his copy of *PCW* from Mr Gerry Taylor, 'a fellow citizen.' Using an Amiga 2000 HD in a VIP Professional worksheet and a substantial amount of intuitive mathematics, he obtained 2.17624??.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.