Join the dots

'On the borders of geometry and arithmetic', or What to do with 'Dotty Paper'; presented by Mike Mudge.

Many readers will have seen the 'DOTTY PAPER' presently used in the teaching and examining of mathematics at an elementary level. GCSE and below... we shall draw on the entire plane squares like those in square graph paper. The plane is thus divided into squares of the same size. The vertices of our squares are called LATTICE POINTS — as on 'DOTTY PAPER'. It may seem that there is little to be said about such points, so evenly spaced on the plane, and further that they are unlikely to invovle any interesting or difficult problems.

However, for 150 years, from the time of Gauss until the present, lattice points have been the subject of various interesting mathematical inquiries. Many problems have been solved but there remain some from the Gauss era, together with many recently posed.

Throughout we shall use the term 'natural number' to mean positive integer, i.e. 1,2,3,4...; it is further hoped that, because most readers will have access to some type of graphical computer output — including the pencil and paper interface! — that diagrams will accompany any arithmetic, or indeed algebraic, results.

Problem A For every natural number, n, does there exist in the plane a circle having in its interior exactly n lattice points? Hint: First show that this is not possible for circles having a lattice point as centre, then show that every two distinct lattice points are at different distances from the point $(2^{1/2}, 1/3)$...

This problem, due to H. Steinhaus and answered in the affirmative, was modified by A. Schinzel to consider only circles whose centres are located at rational points viz. (a/b,c/d) where a,b,c & d are natural numbers having the above property.. the answer is now negated.

Problem B For every natural number, n, does there exist in the plane a circle having exactly n lattice points on its circumference? A. Schinzel, Enseignement Math. (2) 4 (1958) pp71-72 proved the answer to be in the affirmative. But can a computer be used to generate empirical evidence and hence lead to the radius and centre of such a circle, as a function of n?

Problem C For every natural number, n, does there exist in the plane a square containing exactly n lattice points? J Browkin proved that the answer is yes; but how would such a square be constructed for any given n?

Problem D For a given natural number, n, greater than or equal to 3, consider the n^2 lattice points (x,y) where x and y are the natural numbers less than or equal to n. Let R_n denote the set of these points. The problem is to determine the smallest natural number, k(n), for which each subset of R_n having k(n) points contains nine points in three different rows and three different columns. It is known that k(4) = 14, k(5) = 21, k(6) = 27 & k(7) = 34; no other k-values are known to the author... Derive from empirical evidence or otherwise the above test data and attempt to extend it.

Problem E Given that for an arbitrary natural number, n, there exist n lattice points lying on the circumference of some circle and such that the distance between any two of them is given by an integer (W. Sierpinski), construct a computer algorithm to approximate to this set of points for a given n.

Note: S. Ulam posed the very complex and as yet unresolved extension: whether there exists in the plane an everywhere dense set of points, every two of which are at a rational distance from each other. A set of points in the plane is everywhere dense if inside every circle is a point of the set.

SOMETHING TOTALLY DIFFERENT: In the New Scientist of 25 June 1994, page 48, Susan Denham suggested that most years from 1990, 1991 etc. can be represented by the sum of an arithmetic progression a+ (a+1) + (a+2) +.. to n-terms. Consequent upon this, Arthur T Oram of Purley, Surrey sent me a copy of his analysis of the years from 1801 to 2100, showing a failure of Denham's expansion only in the case of 2048. Arthur used his PC to obtain sequences of 1,2,3,5,7,9,11,14 or 15 terms but found none of length 4,6,8,10,12 or 13 and none requiring more than 15: he has no time to investigate further but asks for any confirmation and explanation of these findings from 'Numbers Count' readers.

e.g. 1936 is the sum of the 11 consecutive integers starting at 171 while 2080 is the sum of the 5 consecutive integers starting at 414.

Responses to Problems A through E together with any discussion of 'Denham's Expansion' both for integers in the suggested range and in general may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 October 1994. Any communications received will be judged, using suitable subjective criteria, and a prize of £25 in the form of a book token (or equivalent overseas voucher) will be awarded by Mike Mudge to the 'best' contribution arriving by the closing date. Such submissions should contain a brief description of the hardware used, details of coding, run times and a brief summary of the results obtained; all in a form suitable for publication in *PCW.*

Review of 'Numbers Count' -131- Something Simple, *PCW* March 1994
Among the many encouraging responses this month, Henry Ibstedt's is worthy of special mention: tackling all problem areas fully, his reference to Fibonacci's Problem Book by M Bicknett & VE Hoggatt being particularly interesting. (The Fibonacci Association 1974.) MW Gribble of Abergavenny was attracted to the Fibonacci Numbers and suggested a polynomial expression for the Tribonacci Numbers defined by $u_{n+1} = u_{n-2} + u_{n-1} + u_n$ as a natural extension; his arithmetical requirements were satisfied by a programmable pocket calculator.

Ed Hersom of Thirsk sent a mammoth submission due, he said, to so many interesting topics in one issue! Having been associated with computers since 1949 Ed wrote his first ever game, NIM, for an Agenda with its 20 * 4 character display and programmed in Forth. The very worthy prizewinner is, however, John Pace of 27 High Street, Hamrun HMR-02, Malta, for his investigations of Type I fractions in GW-Basic on a 'trusty' Schneider Euro PC, a single-floppy XT clone. John rejected the use of AMOS on an Amiga — he would not trust it for number crunching. Comments, please.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.