From INT to ZIG & ZAG

Hints for INT(s), presented by **Mike Mudge**. And whither the ZIG-ZAGs?

Two fascinating subject areas, the first strictly numerical, the second combining a simple numerical algorithm with some graphics: designed to appeal to the entire spectrum of *PCW* readers.

THE FLOOR FUNCTION $\lfloor x \rfloor$ is designed to be the greatest integer not greater than x. (This is to be contrasted with the CEILING FUNCTION, $\lceil x \rceil$ defined to be the smallest integer greater than or equal to x.) This function will not be used in the present article and so the notation [x] will be used for the FLOOR FUNCTION.

Alternative definitions of FLOOR(x) are:
(i) the unique integer satisfying x-1 less than FLOOR(x) less than or equal to x, and
(ii) the unique integer with x = FLOOR(x) + y where y is non-negative and less than unity.

In Basic the function INT(x) is precisely FLOOR(x), however in Pascal the function INT(x) 'rounds towards zero' i.e. INTpascal(-3.2) = -3 whereas INTbasic(-3.2) = FLOOR(-3.2) = -3.2 = -4.

Problem 0 Write a program to tabulate ([n1/2] + 1)2 - n and explain the results using n = 1,2,..100.

Theorem (To be proved *only* by those interested!) The power of a prime p which divides n! (n! denoting factorial n i.e. the product 1x2x3x4x...xn) is given by [n/p] + [n/p2] + [n/p3]+...the sum being continued until the terms become 0, i.e. until pr is greater than n.

Problem 1 Write a program, using the above theorem, to factorise n! and also the BINOMIAL COEFFICIENT nCr = n!/(r!(n-r)!).

Test data: 1000! and 1000C353.

NOTE: Multi-precision arithmetic is not needed.

Further discussion of the FLOOR FUNCTION and its applications is to be found in Peter Giblin's recent publication, 'Primes and Programming, An Introduction to Number Theory with Computing', C.U.P. 1993, £12.95 paperback.

ZIG-ZAGS, a fascinating family of closed polygonal arcs.

This topic originates in 'Turtle Geometry' by H Abelson and A diSessa, MIT Press 1980, paperback 1986 — essential reading for any computer users interested in the interaction between mathematics and graphics; it is also discussed in Peter Giblin's book, pp33-35.

The basic rules for the construction of ZIG-ZAGs are as follows:
(i) draw a base line in a horizontal direction of unit length,
(ii) define as input data a different length, L, (any positive number not equal to 1); together with two angles a and b which are to be integer numbers of degrees,
(iii) at the right-hand end of the base line construct a line of length L, also horizontal.. giving a base of length 1+L units.
(iv) now proceed with a sequence of line segments alternately of length 1,L,1,L,1,L and so on making angles measured anticlockwise with the horizontal of a,b,2a,2b,3a,3b and so on. The lines of length 1 are called ZIGS and those of length L ZAGS; Giblin suggests the use of different colours to display these two line-types.

The ZIG-ZAG will always close; indeed, after a number of steps (one step consisting of a ZIG and a ZAG!) given by 360/(360,a,b) where (360,a,b) denotes the Greatest Common Divisor of 360, a & b. Note that if there is no non-trivial common factor then (360,a,b) = 1 and 360 steps are needed.. but how big will the ZIG-ZAG be?

Problem 2 Design and implement a program to construct ZIG-ZAGS for given input triples (L,a,b).

Test data to include: (1,90,-40):(1,62,-60):(1,19,-20):(0.4,45,9):(0.57,32,4):(0.54,35,79):(2,5,6):(2,299,45):(1,175,185).

Problem 3 Estimate the size of the completed ZIG-ZAG and scale the plotting to enable it to fit the screen before commencing the computations.

Giblin page 35 suggests one such estimate with the centre of the screen to have coordinates (.5,.5cot(a/2)) and for the height of the screen to be mod(.5cosec(a/2)) + mod((L/2)cosec(b/2)) where the starting point is at the origin (0,0) and the first ZIG is of length 1. Can this be (a) justified and (b) improved upon?

Now to something different: prompted by Neil Croll, of Allestree, Derby. The problem is to construct a string of 64 binary digits, i.e. 0's and 1's, such that when its two ends are joined to form a simple closed loop, every possible combination of six bits from 000000 to 111111 (i.e. the binary representations of the decimal integers from 0 to 63) will occur in the same direction in the loop.

This problem can clearly be generalised in a variety of ways, including a change from binary to any other number base, a search for the minimal length of loop having the required property and a requirement of multiple occurences such as two binary twos.. three binary threes etc.

Problem 4 Set up an algorithm to solve Neil Croll's problem and generalise in some way. Neil has obtained a solution by trial and error and suggests the use of recurring decimals here?

Responses to some, or all, of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1 December 1994. Any communicaitons received will be judged, using suitable subjective criteria, and a prize in the form of a £25 book token, or equivalent overseas voucher, will be awarded by Mike Mudge to the 'best' contribution arriving by the closing date.

Review of Numbers Count -133- Problem Classification, *PCW* May '94

The subject area produced a disappointing response. But why? The generality of the opening paragraphs was clearly offset by the list of specific (and hopefully easy to comprehend!) problems which followed. Readers are encouraged to read W. Sierpinski on this matter, whether or not they replied to the article, and subsequent reaction to the specific and general problems would be appreciated. Undoutedly the prizewinner is Nigel Hodges or 103 Grange Road, Tuffley, Gloucester GL4 0PT, whose results include:

Problem (a) Upto 2257 - 1 solutions provided by Hans Reisel in Prime Numbers and Computer Methods for factorisation.

Problem (b) Unsolved but formulated as an equivalent polynomial root-finding exercise, my mind goes back to the IBM Symposium on 'Constructive Aspects of the Fundamental Theorem of Algebra', Zurich 1967 (June). Anyone else?

Problem (f) lead Nigel to cite:

"Table of solutions of the diophantine equation y3-x2=k by Lal, Jones & Blundon, Memorial University of Newfoundland 1968, also Math.Comp. vol. 20, 1966, pp322-325."

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles.