Stamp of approval

Something seasonal in the run-up to Christmas — a Postage Stamp Problem. Plus feedback and contributions from readers, presented by Mike Mudge.

This topic first featured in Numbers Count in November 1986. Indeed, the writer first met it in The American Mathematical Monthly, March 1980, where Ronald Alter & Jeffrey A. Barnett define it as follows: An envelope may carry no more than h-stamps and one has available k integer-valued stamp denominations. Given h and k, find the maximal integer n, clearly a function of both n and k in general and thus written n(h,k), such that all integer postage values from 1 to n can be displayed. In addition, find all sets of k stamp denominations satisfying this condition.

Note: The above problem statement is usually modified by augmenting the solution sets of stamps with a stamp of value zero and then requiring that the envelope contains exactly h-stamps.

Example. Given h=2 & k=3 then n(h,k)=n(2,3)=8; the unique solution set is (1,3,4) + (0) and the displayed pairs of stamps range from 1 = 1&0 to 8 = 4&4.

There may be a multiplicity of solution sets, thus n(2,6)=20 and the five solution sets (zero not displayed) are

(1,2,5,8,9,10),
(1,3,4,8,9,11),
(1,3,4,9,11,16),
(1,3,5,6,13,14) and
(1,3,5,7,9,10).

Complete algebraic solutions are rare, however the result n(h,2)=Greatest Integer Less Than or Equal To$(h^2+6h+1)/4$ with a unique solution set

(0,1,(h+3)/2)

if h is odd and two solution sets

(0,1,(h+2)/2) and (0,1,(h+4)/2)

if h is even: date from 1955.

Certain bounds on the function n(h,k) are most interesting; thus if h=k it is known that n(h,h) is greater than or equal to F_{2h} - 1 where F_{2h} denotes the appropriate Fibonacci number from 1,1,2,3,5,8... see, for example, Numbers Count, March 1994, May 1983.

PROBLEMS PS. (i) Determine as many values as possible of n(h,k); target limits on each, say, 13.
(ii) Investigate a possible relationship between n(h,k) and n(k,h).
(iii) What is the multiplicity of solution sets as a function of h & k?
(iv) What is the behaviour of n(h,k) if negative and rational stamp denominations are permitted?
(v) For what values of h & k do symmetric solutions exist? An example of a symmetric (and unique) solution set due to Henrici is

(-1,1,2,4,8,12,16,20,22,23,25)

with range 0 to 48 for n(2,10). ref. The Coins Problem, Part I.Diss.,Diploma in Num.Anal. and Auto.Comput. Corpus Christi College, Cambridge, 1965.

Test data for the above computation are:

n(2,3)=8;n(2,7)=26;n(2,13)=72;n(7,3)=69;n(7,5)=345; n(13,3)=259.

THE RESTRICTED INT FUNCTION

Recall the INT function in BASIC which produces an integer value whatever its argument, INT(x) being the largest integer not larger than x. i.e. Int(3.142) = 3 while Int(-2.013) = -3. Now, in early 1990, Roy Dixon of Bracknell considered restricting this function so that instead of generating all numbers exactly divisible by unity, i.e. the integers, it was to exclude all numbers divisible by an integer, d, say. Trivially, such a function is:

S_n = n + INT((n-1)/(d-1))

thus to skip any numbers divisible by both 2 & 3 we may use

S_n = n + INT(3n/2 - 1) + INT(n/2).

Roy considered further the omission of numbers divisible by 2, 3 or 5, listing two alternatives:

A....4n - 3 +2(1-INT(n/5)(INT(n/2)-2INT(n/4))-2INT(n/5)(INT((n-4)/2)-2INT((n-4)/4)-2INT((n-1)/8)

B....4n - 2 -2INT((n-1)/8) +(4/$3^{1/2}$) Sin ((PI/3)(n-1-INT((n-1)/4)))

PROBLEMS D. (i) Discuss the relative merits of A & B above and investigate alternative F(n) yielding the natural numbers less those having a given set of factors, $(f_1,f_2,...f_n)$. Does this lead to a realistic generation of a table of prime numbers? A sort of analytic "Sieve of Eratosthenes"...
(ii) Investigate the construction of formulae for generating those natural numbers which do NOT have two factors differing by a given integer, D, say.

Hint: Roy has generated the function:

S_n = INT(n + (n +1/4(D+1)(D-3))$^{1/2}$ - 1/2(D-1))

for this purpose where input of 1,2,3,4,... yields 1,2,4,5,6,7,9,... omitting 3 = 3x1 and 8 = 4x2 etc. i.e. numbers having two factors differing by 2 in the case where D = 2.
(iii) In these "Dixon Functions" only integer input is permitted. Extend them so that the behaviour continues for real x replacing n.
(iv) How is (ii) above extended to exclude those natural numbers having factors differing by any of a set of D values?
(v) Find an application, either practical or theoretical, for these functions. I am not sure if Roy has some use for them! since he describes them as "All good fun!"

Now to Michael Priestley of Edinburgh, and PI calculation

Michael calculated PI to 200 places of decimals using a pre-Numbers Count algorithm from *PCW* January 1982 in BASIC on a 64kb Tandy TRS80. However, when he runs this in QBASIC on a Packard Bell II5S it adds a zero after the decimal point and every five digits thereafter. Deleting the 10*H in line 200 (he says) removed the zeros and separated the sequence of numbers into blocks of four, but any block which began with a zero had that zero deleted. What is the explanation and cure for this?

Michael further asks: "Personal Computer World seems to have stopped printing BASIC programs. Can anyone advise on current sources of interesting mathematics-based QBASIC programs?"

Responses to any of the PROBLEM PS & D above, together with helpful advice for Michael Priestley, may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel (0994) 231121, to arrive by 1st March 1995. Any solutions or attempted solutions to the problems received will be judged, using suitable subjective criteria, and a prize in the form of a £25 book token, or equivalent overseas voucher, will be awared by Mike Mudge to the "best" contribution arriving by the closing

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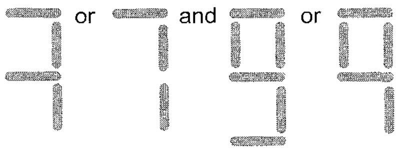
date. Such contributions should contain a brief description of the hardware used, details of coding, run times and a summary of the results obtained; all in a form suitable for publication in PCW. Additionally, readers' comments upon the general, or specific, nature of this month's column would be msot valuable, in particular references to any recent work (either published or unpublished) in the subject areas covered.

WATCH THIS SPACE: There will shortly appear details of the smallest integer which can be expressed as the sum of two cubes in FIVE distinct ways...

FEEDBACK

The combination of Digital Clock Displays and the DONG Currency in Numbers Count, July 1994, produced some interesting responses. The digital displays raised questions of the possible lack of uniqueness of representation, including



for 7 & 9 respectively.

Alan Cox draws readers' attention to a technique for enabling a program to recognise that a particular set of segments represents a digit. A prime number is assigned to each available segment, then each digit has a unique signature (namely the product of these primes, for example) which the program can easily recognise numerically.

Robert Newmark used an Acorn A3000 programmed in BASIC and a sequential numbering of the segments in a clockwise direction starting with the top and finishing in the centre. He, together with numerous other readers, could not resolve figure 1(c)... A response from the originator of the problem, Peter Skuse of Whyteleaf, Surrey, is awaited...

Related Number Bases proved to be of limited interest, but an interesting generalisation from Yogish Sabharwal of 683 Lodhi Road Complex, New Delhi - 110003, India, is worthy of this month's prize. Yogish first considered the general representation of the 3-digit problem (abc) base x = (def) base Y after removing the restriction that the base be an integer! The infinity of solutions for (X,Y) are, naturally, given by the points on a conic-section: however, the implied number theoretic interpretation has now been lost.

Yogish programmed in C, firstly a general-purpose program whch finds "corresponding bases for all numbers within the specified range on the basis of a specific range of bases. Inputs being the initial number and the final number within which all numbers will be tested for results and the initial and final bases within which the bases are to be checked." Secondly, a particular program tests only two input numbers as candidates for bases within a specified range of bases. Copies of coding on request from Mike M.

STOP PRESS The second edition of Unsolved Problems in Number Theory, by R.K. Guy, is now available. Springer-Verlag, August 1994. ISBN 3-540-94289-0. DM69-. This contains not only extensive new material, but corrections and updates have been added throughout the book.

I am anxious to trace the originator of one page of coding headed

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Programming Sample (Copyright
25th March 1994)
BeginPackage ["PrimePacker"] and
including the Ciel [Log[256,#]],
statement.
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This document has been sent to me by *PCW* with no covering communication envelope or other identification — Help!

How irrational can one get?

Recall that the continued fraction expansion of a quadratic surd is always periodic. e.g. $3^{1/2}$ = (1; 1,2,1,2,1,2,...) tables upto $10000^{1/2}$ due to Wilhelm Patz, 1941, and much more recently C.D. Patterson and H.C. Williams, Some Periodic Continued Fractions With Long Periods, Math.Comp.vol.44, no.170, April 1985, pp523-532, include the result that $46257585588439^{1/2}$ has a continued fraction with period 25679652. Now, during a recent conversation with Alan Cox of St Clears he proposed a measure of irrationality associated with any pattern in the continued fraction expansion of an irrational number. Thus e = (2;1,2,1,1,4,1,1,6,..) would be, in some sense, less irrational than PI for which no known pattern exists in the continued fraction. Any comments upon this notion?

PCW Contributions Welcome

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles.

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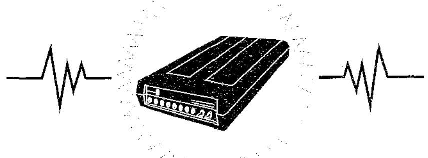

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