On the tiles

Beginners Lesson One (with answer) is followed by a Classic Tiling Problem: mathematics for all, presented by Mike Mudge.

BEGINNERS LESSON ONE

Given any positive whole number, or integer, those positive integers which divide into it exactly are called its FACTORS or DIVISORS. Among these are the IMPROPER FACTORS consisting of the number itself and unity or "one". These latter are of no real interest to us at this time — they are indeed trivial or degenerate cases of factors. Clearly then, given two positive integers, these will have a HIGHEST COMMON FACTOR (HCF) or GREATEST COMMON DIVISOR (GCD). If this object is unity we say that the two given numbers are co-prime; if they are denoted by m & n we write gcd(m,n) = 1.

e.g. 12 = 1x2x2x3 and 35 = 1x5x7 hence (12,35) = 1. If the two numbers m & n are equal then clearly the gcd(m,m) = m, another degenerate.

Problem: how to use a computer to determine the gcd of two given positive integers.

Extension: how to modify this program to determine the gcd of three or more positive integers.

Concern: how to discover the limits on the magnitudes of numbers which a particular program/hardware will handle correctly.

Below is a computer program in PASCAL based on An Introduction to Number Theory with Computing by Peter Giblin, ISBN 0-521-40988-8, 0-521-40182-8. C.U.P. 1993.

PROBLEM (A) Translate the above coding back into algebra and reveal the algorithm (due to EUCLID) for determining the gcd. Implement, either in PASCAL or a simple BASIC translation for example, determine the numerical limit by successive examples (666...666, 777...777) which should produce the answer 111...111. Finally, modify the coding/algorithm as appropriate to handle the gcd of three or more input integers.

```
VAR
        a,b,r: extended;
        (* Use type real if you do not have extended precision, but this *)
        (* limits the number of significant figures to about eleven. *)
BEGIN
        writeln ('type two positive numbers separated by a space');
        read (a,b);
        writeln ('gcd of' , a:0:0, 'and', b:0:0, 'is');
        (* Since a and b change their values during the program, we *)
        (* have to put this here. *)
        WHILE b>0 DO
                BEGIN
                r:=a - b*INT(a/b);
                a:=b;
                b:=r;
                END;
        writeln (a:0:0);
END.
```

Notes: (i) INT in PASCAL rounds towards zero, thus if we allow a negative then a modification is needed at line "r:=".

(ii) The 'formatting statement' a:0:0 ensures that with variable types real or extended the answer is written without a decimal point, i.e. as an integer.

PROBLEM (B) Adopt a different approach to this problem, via the determination and examination or the PRIME FACTOR EXPANSION of each input integer; compare the run times of this with the algorithm given above.. over a range of magnitudes of integer input.. conclusion?

WANG TILES, in theory and practice

Named after American logician Hao Wang, these entities were brought to my attention by Mat Newman of Didcot, Such a tile is a unit square of fixed orientation with one of the figures 1,2,3,4 placed at the midpoint of each edge (these need not be DISTINCT).

We agree to represent such a tile algebraically by Ta,b,c,d): where a is the figure in the topedge and b,c,d are identified clockwise. Given the two types of tile $T_A(1,2,2,1)$ and T(2,1,1,2) we can easily see that the plane can be tiled with adjacent edges of two tiles having the same number... have rows and columns of alternating A's and B's. Mat informs us that no computer program can decide whether such a tiling is possible for arbitrary sets of Wang tiles; but that the question is decidable for certain specific cases.

PROBLEM (C) Investigate the tiling of the plane with the following set of seven Wang tiles:

$T_A(1,3,4,2)$
$T_B(1,2,3,4)$
$T_C(1,4,3,3)$
$T_D(2,2,4,3)$
$T_E(2,3,3,2)$
$T_F(3,1,2,1)$
$T_G(4,1,1,1)$

PROBLEM (D) Investigate the tiling of the plane with a general set of Wang tiles:

(a) with the condition that the numbers on adjacent edges are to be equal.

(b) with some other condition, such as their sum is to be congruent to 1 modulo 2 for example.

PROBLEM (E) For the enthusiast I think! Investigate the tiling of the surface of a cuboid, say lxbxh units, with appropriate matching at all of the edges.

Responses to any of the problems above may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel 0994 231121, to arrive by 1st May 1995. Any solutions or attempted solutions received will be judged using suitable subjective criteria, and a prize in the form of a £25 book token, or equivalent overseas voucher, will be awarded by Mike Mudge to the "best" contribution arriving by the closing

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date. Such contributions should contain a brief description of the hardware used, details of coding, run times and a summary of the results obtained; all in a form suitable for publication in *PCW*. Additionally, readers comments upon the general, or specific (i.e. BEGINNERS LESSONS?) nature of this month's column would be most valuable. In particular, references to any recent work (either published or unpublished) on WANG TILES.

Please note that material can only be returned if a suitable stamped addressed envelope is enclosed.

FEEDBACK, both POSITIVE and NEGATIVE

The difficulty of solution of a mathematical problem is often said to be inversely proportional to the length of its statement: Dr. Reg Silvapulle of Edgware asks: "I just wonder whether it is possible to find four positive integers a,b,c,d, such that: c^2 = a2 + b2 + ; d2 = a2 - b2 ?"

Eric Adler of 'The proof is in the Pi' fame, *PCW* January 1995, pp467-8, finds that UBASIC deals very adequately with the factorisation problem H_1 from December 1994, while many other well known mathematical packages do not... Experiences on the factorisation of integers greater than 10^{30} would be most welcome.

Review of Numbers Count -137- September 1994: JOIN THE DOTS etc.

The etc. has proved to be the most attractive element of the column, generating record response. Space permits only a sample of the various types of response, with apologies to the many respondents whose approach is not mentioned.

Bob Smith of Ytteren, Norway, a self-confessed "computer slave" since 1940, was inspired by the reference to 2048 (recognised as a power of 2) and used QuickBASIC on an Acer 915P 286 PC running at 10MHz with the Norton Commander operating system on DOS 4.01 to search other intervals and reveal — at least empirically — that powers of 2 are the only exceptions. Bob thought to offend mathematicians by invoking ancient history with all years BC being given negative signs to obtain 2048 = -2047-2046....0+1..+2047+2048.

Eric Spielman of Loughton expressed the base of each sequence (b) in terms of the year (y) and the number of terms (t): thus, b = y/t - (t-1)/2 and then tested if b is a positive integer, necessary to give a valid solution, he observes that for most numbers there are many solutions. For instance, the year 1980 has 11 solutions starting with 3 terms (base 659),5,8,9,11,15,24,33,40,45 and finally 55 terms (base 9).

Franz-Josef Rosselli of Bonn, Germany, provides a very elegant analysis of the sum S(n,k) = n + (n+1) +...+(n+k) = 1/2(k+1)(K+2n) where one of the factors is odd, and continues to establish a 1-1 correspondence between the odd divisors of a number N and its Sylvester series S(n,k).

Henry Ibstedt's equivalent result that any natural number, $N = 2^t p_a^a p_b^b \ldots p_z^z$ can be expressed as a sum of consecutive natural numbers in exactly c(N) = d(N)/(1+t) - 1 ways; where d(N) the number of divisors of N is equal to (1+t)(1+a)(1+b)....(1+z) was extremely well presented with supporting computation in Visual Basic for a dtk 486/33 computer with an HP IIP laser printer.

Andrew Jones of Harlow drew an analogy with Gauss (age 10) summing 0+1+2..+100 and did support computing on his "trusty Tandon 386-25SX using only Ami Pro." Robert Newmark of Sunderland introduced the irrelevance that prime numbers are composed *only* of their mid-pairs and no other series. Paul Weston of Birmingham was attracted by the problem since it is a case where "some analytic work with pencil and paper saves a lot of computing time." [*Doesn't it always? Mike M.*] Echoed by David Ashley of Swansea, whose analysis "confirms the need, recently expressed by one of the major Engineering Departments, for students to eschew an instant use of numeric methods with calculators and computers when simple algebraic reasoning will often produce an answer much more easily."

After much soul searching the very worthy prizewinner is Calum Grant, of 348 Sarehole Road, Hall Green, Birmingham B28 0AQ, whose theoretical presentation is supported by work in Turbo C++ 3.0, on a 486DX/33..not using much computer time: additionally Calum has some results, at least in sketch form, relating to the DOTTY PAPER. Other readers wishing to contribute in this area are invited to contact Calum directly or myself.

PCW Contributions welcome

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles.

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