In your prime

Beginners Lesson Two (with partial answer) together with a selection of problems sent in by readers; presented by Mike Mudge.

BEGINNERS LESSON TWO: A PRIME NUMBER is defined to be a positive integer which is only divisible exactly by itself and unity. (One.) Thus 2 is the only EVEN PRIME, all other even numbers being divisible by 2; the list of PRIME NUMBERS BEGINS 2,3,5,7,11,13,17,19,...

It is well known that the number of PRIME NUMBERS is infinite.

Proof due to Euclid Suppose that $p_1 = 2$ less than $p_2 = 3$ less than... less than p_r are all the primes. Let $P = p_1p_2...p_r + 1$ and let p be a prime dividing P; then p cannot be any of $p_1,p_2,...p_r$ otherwise p would divide the difference $P - p_1p_2...p_r = 1$, which is impossible. So this prime p is still another prime and $p_1,p_2,...p_r$ would not be all of the primes.

Algorithm for listing all PRIME NUMBERS upto a given N

Consider only the odd numbers upto N (2 can trivially be added to the list at the beginning) for any such number, n say, divide in turn by each odd number less than the square root of n. If no factors are found then n is PRIME.

The computer program in Fig 1 is written in PASCAL and based closely upon that given on page 38 of An Introduction to Number Theory with Computing by Peter Giblin, ISBN 0-521-40988-8 and 0-521-40182-8. CUP, 1993.

PROBLEM (A): (i) Implement the coding in Fig 1 and hence output all PRIME NUMBERS less than 5000; there are 669 of these.

(ii) Modify the above algorithm to divide by all of the primes less than square root of n (rather than all of the odd numbers less than square root of n; satisfy yourself that this is an adequate procedure. Compare the efficiency, measured in terms of run time, of the two algorithms.

(iii) Now omit the listing of the prime numbers but simply count them, to obtain PI(N) the number of prime numbers less than or equal to N; verify and extend the table in Fig 2.

Fig 1 Prime Number Program In Pascal

```
VAR
n,q,max:extended;
qdividesn:boolean;
BEGIN
write(' Max = ? ');
        (*Prompt for N: all primes less than or equal to N are printed *)
readln(max);
write('2');
n:=3;
WHILE n lessthanorequalto max DO
        BEGIN
        q:=3;
        qdividesn:=false;
        WHILE (q*qlessthanorequalto n) AND NOT(dividesn) DO
                BEGIN
                qdividesn:=(n=q*INT(n/q));
                q:=q+2;
                END;
        IF NOT(qdividesn) THEN write(',',n:0:0);
        n:=n+2;
        END;
END.
```

Fig 2 Verify And Extend This Table

N/1000	=	1,	2,	3,	4,	5,	6,	7,	8,	9,
PI(N)	=	168	303	430	550	669	783	900	1007	1117

PROBLEM (B): Modify the above program to print out only those primes which differ from the previous one by a given gap factor, g, which is to be input. Find the smallest primes (consecutive) which differ by 10*s where s = 1,2,3,4,...

PARAPRIMES: A study introduced by Charles Lindsay of Bangor, Co. Down

Consider a PRIME PAIR, i.e. two primes differing by 2. e.g. 5,7 or 11,13, the intermediate even number is a PARAPRIME and in particular is to be called an INTERPRIME. Notice that the first interprime is 6 and the second 12 = 6*2.

Question L1. What is the set of multipliers $m_1,m_2,...$ by which one can "leap" from a given prime pair to another prime pair? Experimenting, Charles found that in quite a lot of cases m = 6. Factorisation of some of the interprimes revealed such results as $1620 = 2^2*3^4*5$ and 1698 = 2*3*283 hence we write:

(5-<u>6</u>-7) *283 = (1697-<u>1698</u>-1699

Here the 283 has leapfrogged over a large number of prime pairs; note it is itself prime.

Question L2. Which of the m are prime? The primes turn up in a most irregular order as one might expect.

Question L3. Does every prime appear in the list of the m_i?

Question L4. If there is a last prime pair, how does the whole system shut down?

Question L5. Consider (1-<u>2</u>-3) * 6^2 = (71-<u>72</u>-73), ignoring the legality of using 1 here we are dealing with m^2: what about m^3, m^4...? Note that the primes which follow 283 are 397, 467 and 577 and 6* — these are also interprimes!

LEGENDRE'S CONJECTURE, with acknowledgement to George Sassoon of Warminster and to Adrian Berry in the Daily Telegraph of 1 Jan & 15 Jan 1995.

$A^3/B^3 + C^3/D^3 = 6$ was thought by Legendre to have no positive integer solutions, with A, B, C & D mutually prime and all above 100. There is a solution A=17; C=37: B=D=21.

However, Kevin Brown of Washington, USA, found a solution with around 100 digits for each of A,B & C=D while Dr. John Cohn found a solution set with around

Fig 3 Ibstedt/Guy Comparison

	Ibstedt	Guy
Number of Congruent Numbers	167	352
Number of Non-Congruent Numbers	126	231
Number of Numbers with Repeated Factors	392	392
Number of Unsettled Cases	315	25

1600 digits per number. The solution of Kevin Brown, see the Daily Telegraph of 15th January, reveals A divisible by 17, C by 37 and B=D by 21. Prompting George to ask:

Question GS: Are there any solutions where B $\neq$ D, are there any smaller solutions than those of Kevin, are all the solutions multiples of 17,37 and 21 respectively?

What happens if the numerical parameter is changed from 6 to another positive integer?

Responses to any of the above problems and questions may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthen, Dyfed SA33 4AQ, tel 01994 231121, to arrive by 1st June 1995. Any complete or partial solutions received will be judged using suitable subjective criteria, and a prize in the form of a £25 book token, or equivalent overseas voucher, will be awarded to the "best" solution. Such contributions should contain a brief description of the hardware used, details of coding, run times and a summary of the result obtained; all in a form suitable for publication in *PCW*.

Feedback from readers

The second edition of Unsolved Problems in Number Theory by Richard K. Guy, Springer-Verlag 1994, ISBN 0-387-94289-0, 3-540-94289-0 is now available. I cannot recommend this work too highly, a must for all empirical number theorists; giving the state of the art in a vast range of investigations. Does any reader know if a revised edition of David Wells' Penguin Dictionary of Curious and Interesting Numbers is due to be published?

A variation on the standard "minimum set of weights" problem has reached me from Clive Tooth via Alan Cox. Here both scale pans may be used, there is a set of seven weights but only three from the set can be used in any given weighing. Some time ago, May 1994, Alan discovered the set 13,22,26,29,30,31 & 32 which enables weighing of upto 93 units. Can any reader shed some light, either in the form of a "better" set or some general theory?

A new publication has arrived on my desk, OCTOGON Mathematical Magazine, of The Laszlo Zsido Mathematical Society, ISSN 1222-5657, published by Fulgur Ltd, Brasov, Romania. Some of its open questions and problems may be of interest to Numbers Count readers.

Review of Numbers Count -138- October 1994. Extensible sets, congruent numbers and a miscellany from the Archimedeans.

Gareth Suggett refers to H.J. Godwin in Math.Comp. Vol. 32, pp293-295 1978 in relation to CONGRUENT NUMBERS. Godwin finds (x,y) for which $x^2 + py^2$ and $x^2 - py^2$ are both squares and Gareth implemented the routine for p of the form 8n+7. Results ranged from:

p(x,y) = 23(905141617, 144613560) to 919(99095140556255203086160602497 80900096,311030032483489146841867 803030681600). Gareth goes on to report that with regard to congruent numbers under 1000 he still has a very incomplete list with gaps under 100 of the form 8n + 5,6 or 7 at 29,37, 38,53,61,62,77,78,87,93,94 & 95 even taking into account the results obtained by Godwin's method.

Now to Henry Ibstedt of 7 rue de Sergent Blandan, 92130 Issy les Moulineaux, France, who is this month's very worthy prizewinner. He found congruent numbers to be "very interesting" and the results "so surprising"; visiting Jussieux University and the Henri Poincare Institute to reveal a host of related literature. Undoubtedly the most up to date reference is in R.K. Guy's Unsolved Problems in Number Theory, but the most significant observation is that the 18 "new" congruent numbers found by Alter, Curtz and Kubota since their initial article in the Proc. 3rd Conf. on Combinatorics, Graph Theory and Computing in 1972 are 101,143,159,181, 215,287,349,479,511,533,535,719,731, 821, 879, 901, 935 and 943. Upto 1000 it is interesting to compare Henry's findings on a DTK 486/33 computer with the Guy "state of the art" (Fig 3). So there is a great deal still to be done in this area...

PCW ContributionsWelcome

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future **Numbers Count** articles.