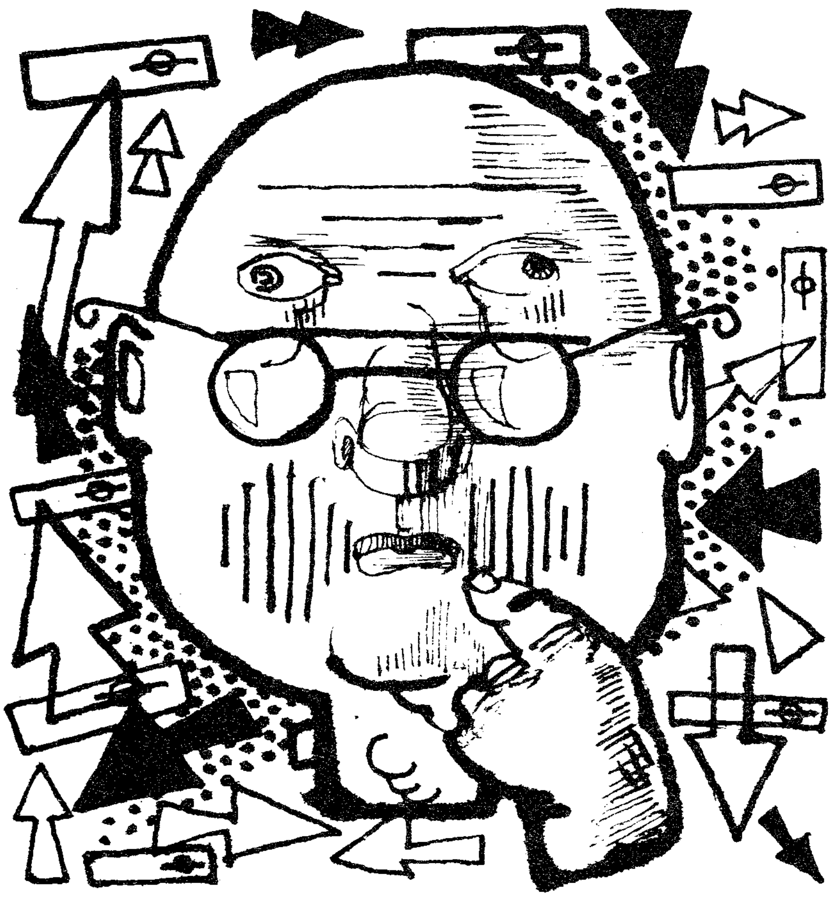
# Top of the class

**Some sequences of Smarandache contrasted with a permutations problem relating to class/student allocation; presented by Mike Mudge.**

Some Sequences of SMARANDACHE... contributed by J Yan of Tucson, Arizona.

(1) The Smarandache Consecutive Sequence

1, 12, 123, 12345, 123456,... What fraction of this sequence is prime? Generalise this problem to any number base B. References: Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache. Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA, tel: (602) 965-6515 (Carol Moore librarian), email: ICCLM@ASUACAD.BITNET.



(2) The Smarandache Digit Sequences

General Definition: in any numeration base B, for any given infinite integer or rational sequence S1, S2, S3,... and any digit D from 0 to B - 1 we define a new integer sequence which associates with S1 the number of digits D of S1 in base B, with S2 the number of digits D of S2 in base B etc.

e.g. Considering the prime number sequence in base 10, then the number of digits, says 1, of each prime number following their order is: 0,0,0,0,2,1,1,1,0,0,1,0... (The Smarandache digit-1 prime sequence, the penultimate entry in the sub-sequence quoted corresponding to the one 1 in "31").

Similarly, the Smarandache digit-0 factorial sequence begins: 0,0,0,0,0,1,1,2,2,1,3... whilst the Smarandache digit-5 n*n sequence begins 0,0,0,1,1,1,1,0,0,0,... the last 1 in this subsequence representing the 5 in $7^7$ = 823543. References include: Florentin Smarandache, "Only problems, not solutions!", Xiquan Publishing House, Phoenix-Chicago, 1990, 1991, 1992, 1993; ISBN: 1-879585-00-6, Unsolved Problem 3, p.7.

(3) The Smarandache Construction Sequence

General Definition: in any numeration base B, for any given infinite integer or rational sequence S1,S2,S3,... and any digits D1,D2,D3,...Dk (where k is less than B), we define an integer sequence such that each of its terms Q1 less Q2 less Q3... is formed by these digits D1,D2,D3...Dk only (all these digits are used) and matches a term Si of the previous sequence.

e.g. Consider the base 10 prime number sequence starting 17, 71,... called the Smarandache digit-1-7-only prime sequence. The Smarandache digit-0-1-only multiple of three sequence begins 1011, 1101, 1110, 10011, 10101, 10110, 11001, 11010, 11100,... References include Arizona State University cited above, and "Only problems, not solutions!"

(4) The Smarandache Symmetrical Sequence

11, 121, 1221, 12321, 123321,....... 12345678910111213121110987654321.. Florentin Smarandache asks, how many prime numbers are there in this sequence? NOTE: In its most general form the Smarandache Symmetrical Sequence is considered in base B (Radix B arithmetic). Reference: "The Encyclopedia of Integer Sequences" by NJA Sloane and S Plouffe, Academic Press, 1995; on email: superseeker@research.att.com

PCW Photography by Jake Abrams

(SUPERSEEKER by NJA Sloane, S Plouffe, B Salvy, ATT Bell Labs, Murray Hill, NJ 07974, USA).

Problem Y (an.) (a) Investigate the Smarandache Consecutive & Smarandache Symmetrical Sequences in a general number base, say B, examining the occurrence of both PRIMES and indeed any other special types of integer that may be considered appropriate, e.g. Fibonacci, Triangular, Binomial Coefficients etc.

(b) The Smarandache Digit & Smarandache Construction Sequences are clearly (very) open-ended. Implement the examples given, in the first instance, and carry out a frequency count of entries in the associated sequences. Consider the possibility of modelling this algebraically. Then extend the philosophy (i.e. the general definition of these sequences) in any way that appears to be either natural or interesting and repeat the analysis. The above problems appear to me to be of a most abstract and in some sense unnatural type (COMMENTS ON THIS OBSERVATION PLEASE!). While realising that today's Pure Mathematics may be tomorrow's Applicable Mathematics, I wish to contrast with a CLASS/Student allocation problem posed by Neil Charlton.

A PERMUTATION PROBLEM from the real world

If I have S unique items and I wish to group them in G groups, how many possible combinations are there? For example, if I have 5 items (S=5) called a, b, c, d, e some permutations for three groups (G=3) are

a b cde; a bc de; abc d e;

(Note: a b cde is strictly equivalent to b a cde and is only counted as one permutation.) Assuming that it is possible to calculate P as a function of S and G, Neil asks for an (efficient) algorithm which would generate all of the sequences. (He suggests the use of for-next loops.) Typical output he suggests for the above example are 12333 — implying that A is in the first group, B in the second etc.

This appears to be a straightforward application of permutation theory, but since it originates in a "real world" situation we ask:

Problem NC Design and implement an efficient algorithm for the solution of the above problem; particular attention needs to be given to the format of the final output... and, to relate this to the typical difficulties with Numbers Count problems, specify the integer limitations on the algorithm supplied for a given integer length arithmetic implementation. Can the count, P, be represented asymptotically for large S and/or G? Is there any known practical application of this asymptotic situation? Responses to either or both of the above problems may be sent to: Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed SA33 4AQ, tel 01994 231121 to arrive by 1st August 1995.

Any complete or partial solutions received will be judged using suitable subjective criteria, and a prize in the form of a £25 book token, or the equivalent overseas voucher, will be awarded by Mike Mudge to the "best" solution arriving by the closing date. Such contributions should contain a brief description of the hardware used, details of coding, run times and a summary of the results obtained, all in a form suitable for publication in *PCW*. Additionally, readers' comments upon the general, or specific, nature of this month's column would be most welcome. In particular, the contrast between between the applied and the abstract problem... which type do YOU prefer? References to any recent work on the Smarandache Type Sequences (either published or unpublished) would be greatly appreciated. Please note that material can only be returned if a suitable stamped addressed envelope is provided.

**Feedback from readers**

The introduction of the BEGINNERS START HERE feature has not, at this time, generated a significant response. Is this what readers want? Is there a different format of "beginners" articles which would be likely to prove popular? Please suggest any subject areas, or specific problems that you would respond to...

There has been pressure from several quarters to write something about the National Lottery. In the writer's opinion there is more than sufficient information and misinformation already available. However, if this topic is of interest and felt to be relevant to Numbers Count please let me know.

Following up on Leisure Lines, *PCW* April 1995, Alan Cox requests the most efficient algorithm for establishing that all of the digits of a given integer are distinct. This is easily done by eye, but "How do we do that?"

**Review of Numbers Count -140- December 1994: A Christmas Miscellany**

As expected, a wide variety of problems attracted a wide variety of responses, including a creditable one from Lahousse Gustaaf of Grimbergen in Belgium using Turbo-Basic on a PC 1512 Amstrad (6MHz) with 640kb and a hard disk of 20Mb. Lahousse attempted the $G_1$ problem with an array size limitation of 8000. He devoted about three hours "to writing two programs, for testing and running them" and came up with some valuable output. He goes on to say, regarding $H_1$: "I cannot buy manufacturer's packages (they are not cheap) so I wrote programs for multiplying and dividing two numbers with more than 4,000 digits, and they are to extend to 10,000 and more digits for a bigger PC." He also goes on to observe 29 = 3.11 - $2^2$...

Problem $H_1$ has attracted much interest. Gordon Bird of Street obtained the required factorisation in three hours on an Amstrad 1640 using version 3.0 of Derive. Michael Cohen used Maple V on a 486DX/33 and solved it in nine minutes, while Alan Cox used UBASIC's MPQSX program on a 12MHz machine and obtained the answer in less than two minutes. Very extensive solutions were received from Henry Ibstedt in Paris and Nigel Hodges in Gloucester; the latter using a remarkable mixture of theory and practice and fining among many other things the first non-trivial solution of $y^2 - 1549 - x^2 = 1$ where y has 71 digits and x a mere 70.

However, the very worthy prizewinner this month is David Broughton, of 17 Golden Ridge, Freswater, Isle of Wight, PO40 9LE. Using an IBM PC compatible with a 386DX/33MHz CPU without a maths co-processor, running Desqview under PCDOS 6.1 and using a 4DOS version 4.0 as the command line interpreter, David concentrated his efforts on Problem $N_2$: finding delay taps on a shift register; although he concludes that "An efficient solution to the converse problem of determining the delay from a given tapping pattern has not been solved". Technical detail including the overcoming of the absence of a parity funciton from high-level languages are discussed in precise terms, full listings are supplied. Details on request from David... I hope!

**PCW ContributionsWelcome**

**Mike Mudge** welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future **Numbers Count** articles