The luck of the Irish

Some Gaelic mathematics and other interesting stories, including a review of January's postage stamp problem, presented by Mike Mudge.

An Irish pattern-makers problem!

From Letterkenny, Co. Donegal, a letter which concludes, "Is mise le meas, Cólm O Cinnéide." poses the following:

"Imagine a string of numbers chosen from (1,2 or 3). For example,

1232123132331132331323323132331323213 132313231331

In the string above, 3 occurs beside 3 which is really a bit boring. From this, suppose you apply the rule 'if a substring x is beside substring y' then x does not equal y. That is in a valid "superstring" (IRISH PATTERN!). 1 would not occur beside 1, 1213 would not occur beside 1213, 12131321 would not occur beside 1, 1213 would not occur beside 1213, 12131321 would not occur beside 12131321... etc.

The most formal terms I [Colm] can think of putting this in is:

(#x = #y) and (x~y) implies (x<>y) i.e. IF the length of a substring x equals the length of a substring y AND x is contiguous to y THEN x does not equal y.

Some example strings are:

(a) 121323121312321
(b) 232131232123132
(c) 313212313231213

If only two digits are allowed, the following strings are all that happen:

0; 1; 01; 10; 010; 101; "

PROBLEM Colm. Enumerate all the valid "superstrings" for the set of three characters (1, 2 or 3). Extend this algorithm to the set of n-characters (x1, x2, x3,....xn) commenting, if possible, on its efficiency.

Possible Hint: Is this related to the Wang Tiling of the infinite plane discussed in *Numbers Count, PCW* March 1995?

The Mull Factoring Group

The Cunningham Project using digital computers to find the factors of certain large numbers is organised by Professor Sam Wagstaff in the Department of Computer Sciences, Purdue University, West Lafayette, IN 47907, USA, tel 317 494 6010, fax 317 494 0739.

Tables of factorisations are published in the book: Factorizations of $b^n \pm 1$ for b = 2,3,5,6,7,10,11,12 upto high powers Ed. Brillhart et al and published by the American Mathematical Society in various editions as vol. 22 of their contemporary mathematics series.

Now, the Scottish arm of this project is led by George Sassoon of Ben Buie Lodge, Lochbuie, Isle of Mull, Argyll PA62 6AA, and it also has a Wiltshire subgroup. It has factorised about a dozen numbers greater than 10^{80} using several computers, and Professor Yuji Kida's suite of PPMPQS programs is about 10^{101}. The biggest number factorised to date (19/4/95) by the group had 98 digits and took several months of spare computer time on IBM-compatible computers. George asserts that now with 486s and a Pentium it would be a lot quicker and cites his own factorisation of an 80-digit number in about three days using a single 90MHz Pentium Dell.

Some Notation. 5,213+ denotes $5^{213}+1$, 3,419- denotes $3^{419}-1$. The suffixes L and M denote the algebraic factors of bn±1. Thus 12,321+ has algebraic factors 12,327L and 12,327M when n+3 mod 6, k = (n+3)/6, $L=12^{2k-1} - 2^{2k}-13^k + 1$ and $M = 12^{2k-1} + 2^{2k-1}3^k + 1$. Similarly for some other b & n. Duplication of work is posing a problem, e.g. the German group factored 12,327M just before MullFac but Wagstaff is trying to co-ordinate efforts to prevent this happening.

PROBLEM GS-MullFac. Attempt to rediscover the wheel by finding the 37-digit prime factor of 5,575L and also the 44-digit prime factor of 11,219+.

If possible, complete the factorisation of these 92 and 94-digit numbers respectively... THEN CONTACT GEORGE SASSOON for instructions and advice as to how best extend your experience of this 'very infinite'! area of exploration.

Incidental information: George Sassoon has recently ordered a 28.8 kbaud modem and is about to venture onto the Internet — comments welcome.

An Update on Multi-Perfect Numbers

Jason Moxham of Southampton, email JLM194@SOTON.AC.UK has found a total of 1067 multi-perfects. Recall that a number is multi-perfect of degree n if it is equal to n times the sum of its factors. (Including the improper factors of unity and the number itself.)

$120 = 2^3 \times 3 \times 5$ with factors 1,2,3,4,5,6,8,10,12,15,20,24,30,40,60 & 120.

These sum to 360 i.e. 3x120 which is therefore triperfect or multiperfect of degree 3.

David Wells; Penguin Dictionary of Curious and Interesting Numbers, suggests that over 500 multi-perfects are known with degrees upto 8. Even with his extensive calculations, Jason has yet to find a multi-perfect of degree 9. The counts of degree n are C_n, Fig 1.

PROBLEM JM. Reproduce and extend if possible the results in Fig 1; are there any theoretical results relating to the existence

Fig 1 How the counts of degree n are C_n

n	3	4	5	6	7	8	
C_n	6	36	63	228	413	321	giving a total of 1067.

of multi-perfect numbers of degree greater than 8?

NOTE: Jason is about to (31/3/95) experiment with 'PARI', a number theory package supposedly better (in some sense?) than UBASIC. Has any reader experience of this software which they would be willing to share either with Jason or, indeed, with all *Numbers Count* readers?

An Investigation of Legendre's Conjecture, CJ1, May 1995

Here readers were asked to investigate solutions of the Diophantine Equation: $x^3 + y^3 = Az^3$ in the special case A = 1. Much response has been received and Nigel Hodges of Cheltenham has given much food for thought. The fundamental reference for much *Numbers Count* material is "The History of the Theory of Numbers", a fascinating trilogy; in vol. 2 from page 572 onwards, the above equation is discussed. Dickson stated that there are no rational solutions for A = 3, 5 or 6. (In fact, for A = 6 there is the solution (x,y,z) = (37,17,21).)

J. Prestet provided the following wonderful result:

if (x,y,z) is a solution, then so is (X,Y,Z) where $X = x(2y^3 + x3)$, $Y = -y(2x3 + y^3)$, $Z = z(x^3 - y^3)$.

Starting with A = 6 and (37,17,21) Nigel generates an all positive solution with X & Z having 26454 digits and Y having 26453 digits. Pepin has shown that there are no solutions for A = 14,21,38,39,57,76 & 196; whilst Lucas shows that there are only solutions when A is of the form $ab(a+b)/c^3$ for integer a,b & c.

PROBLEM CJ/NH. Investigate the obvious generalisations of this equation, i.e. with additional terms on the lefthand side and/or higher equal/unequal powers.

NOTE: Summaries of results for 'small' integers may provide an insight into some underlying theory... MULTIPRECISION INTEGER ARITHMETIC is NOT NEEDED to contribute to this problem.

A problem of random numbers

In the *Daily Telegraph* for Saturday 22nd April, Adrian Berry (science correspondent) reports on a letter in *Nature* by Robert Matthews, science correspondent of the *Sunday Telegraph*. This letter refers to "a mathematical theorem which says that if any group of numbers if chosen at random, there is approximately a 61% chance that it will not have any factors in common". Now, in Mathematical Recreations and Essays by W.W. Rouse Ball and H.S.M. Coxeter, 1947 reprint page 349, it is stated that if two positive integers are chosen at random, the probability that they will have no common factor is $6/\pi^2$. Mr. Matthews applied the above principle to "the numbers created by the positions of the one hundred brightest stars which are placed at random in the sky". The results showed that 61.3% of such numbers had no common factor and if the above theory is applicable this led to an estimate for π of 3.13, correct to within less than 0.4%. This agreement appears to be better than would be expected on a simple intuitive argument, for a sample of one hundred taken from an infinite population.

Matthews concludes: "This shows that the language of the universe is that of higher mathematics" and further "this supports the belief of the ancient Greeks that numbers are at the root of everything in the universe".

Problem PI*ES. Sample integers from a variety of distributions whch appear to be random/pseudo-random/quasi-random. Test for the existence of common factors and use the fraction, F, of these numbers with no common factor to construct estimates of π called π_e using π_e = SQRT(6/F). Address the question, are these estimates within the 'expected' range, given the sample sizes used?

Does this approach provide an additional test for randomness? If so, how can it be efficiently carried out?

Complete, or partial, responses to the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthen, Dyfed SA33 4AQ, tel 01994 231121, to arrive by 1st September 1995. Any complete or partial solutions received wil be judged using suitable subjective criteria, and a prize in the form of a £25 book token, or the equivalent overseas voucher, will be awarded by Mike Mudge to the 'best' solution arriving by the closing date. Such contributions should contain a brief description of the hardware used, details of coding, run times and a summary of the results obtained; all in a form suitable for publication in *PCW*. Additionally, readers' comments upon the general or specific nature of this month's column would be most welcome: in particular, the balance between research projects requiring multi-precision integer arithmetic and those which do not. THIS COLUMN BELONGS TO YOU, the READERS. Those with multi-precision facilities like to show them off. However, those who do not have them can still make a very valuable contribution to empirical number theory by using their programming skills and conducting an orderly and structured investigation to problems such as CJ/NH above.

Review of Numbers Count -141- January 1995: "Stamp of Approval", accompanied by the restricted INT FUNCTION

Was the poor response, numerically, NOT in quality!, associated with the season of this particular article (Christmas)? Previous experience with *PCW* readers suggests the latter. Those readers interested in the postage stamp problem may either refer to 'Algorithms for Computing the h-Range of the Postage Stamp Problem' by Svein Mossige, Mathematics of Computation, Vol. 36, No. 154 April 1981, or contact the University of Bergen, Bergen, Norway, in particular Christoph Kirfel of The Mathematics Institute, SVD. B, 5014 Bergen, Norway, for the latest results.

Suffice it to say that one of our regular readers, Gareth Suggett, extended his results to n(6,4) = 114 with solution set 0 1 4 19 33. Any advance on this, please? Ernst S. Selmer produced a two-volume document at Bergen in 1986... available on loan from Mike Mudge.

The RESTRICTED INT FUNCTION produced some interest, however. After much soul-searching (using an efficient search algorithm!) the prize must be awarded to its originator, Roy Dixon, of 119 Bullbrook Drive, Bracknell, Berkshire RG12 2QR. Thank you, Roy, for providing this sort of stimulus with such foresight.

Further results obtainable on request to M.M. or R.D.

Cry for help!

What are Dimitrov Wheels or, indeed, Serotic Wheels, and how do they adjust one's chances of winning the National Lottery? Replies to M.M. or to David Brake at *PCW*.

Mike Mudge

PCW Contributions Welcome

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future *Numbers Count* articles.