Base Solution

The Smarandache Function revisited, plus a reader's miscellany, by **Mike Mudge**.

Regular readers of Numbers Count will be familiar with the definition of the Smarandache Function. But it is reproduced here in recognition of the widespread interest that it has attracted.

The Smarandache Function, S(n), is defined for positive integers, n, as the smallest integer such that S(n)! is divisible by n.

Note: n!, read as factorial n, denotes the product of all positive integers up to and including n: further 0!=1 by definition.

Problem A

Due to Charles Ashbacher, Cedar Rapids, IA 52401, USA.

For what triplets n, n+1, n+2 does the Smarandache function satisfy the Fibonacci reccurrence:
`S(n)+S(n+1)=S(n+2).`
Charles has found the solutions:
`n=9,119,4900,26243,32110,64008,368138 and 415662.`
His comment: "I am unable to discern a pattern in these numbers that would lead to proof that there's an infinite family of solutions." Perhaps another reader can?

Problem B

Due to I M Radu, Mathematical Spectrum, vol 27, no 2.

Show that, except for a finite set of numbers, there exists at least one prime number between S(n) and S(n+1).
Among the finite set of numbers referred to above are:
`224-225, 2057-2058, 265225-265226 and 843637-843638…`

Factors of the last two pairs of numbers incude prime pairs:
`(103,101) & (151,149)`
respectively… does this provide the clue in the search for additional values?

Problem C

Due to A Stuparu, Valcea, Romania.

Consider numbers written in Smarandache Prime Base or Smarandache Square Base: how many are prime, how many are perfect powers? (In particular, perfect squares and perfect cubes.) e.g. 101 in Smarandache Prime Base 1,2,3,5,7,11… is equal to $1.3+0.2+1.1=4_1$ and is therefore not prime in The Smarandache Prime Base, but is prime in the decimal base.

Note: Academic Press will publish *The Encyclopedia of Integer Sequences* by N.J.A.Sloane and S. Plouffe in 1995; a disk with the sequences will also be available.

Feedback — positive and negative

● Prime Pairs: i.e. n±1 which are both prime, have attracted much attention recently. Tony Forbes of Surrey, using a 33MHz 486 microprocessor, upgraded to 100MHz, programmed in a combination of Yuji Kida's UBASIC and PC assembler to discover a pair of the order of 10^{4662} (20th July 1995) — a result worthy of publication in the "learned journals"! — he has also made a study of prime k-tuplet patterns up to k=14, details from M.M.

● Email Addresses. I would like to thank George Sassoon for the following: "A list of email addresses of people interested in computational number theory compiled by Andrew Odlyzko is available on amo@research.att.com or from George at New Farm, Tytherington, Warminster, Wiltshire, BA12 7AA. Tel: 01985-840205."

● Factorisation of "LARGE" Integers. Paul C Leyland of Aylesbury wishes *PCW* readers to know (4th June 1995) that the Cunningham files are available for anonymous ftp at ftp.ox.ac.uk in directory/pub/math/cunningham. His email address is pcl@oucs.ox.ac.uk and he would be pleased to hear from other people interested in factoring. Sam Wagstaff's address is ssw@cs.pudue.edu.

LIP multiprecision package written in C and supporting arithmetic on integers of arbitrary size, written by Arjen Lenstra, is available at no cost for non-commercial use from ftp.ox.ac.uk in the /pub/math/freelip directory in source code. But, Paul warns: "An hour's thought will often make more progress than a month's computation."

Complete or partial responses to these Smarandache problems, along with work on prime k-tuplets and related problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthen, Dyfed. Tel: 01994 231121, to arrive by 1st January 1996. Any complete or partial solutions received will be judged using suitable subjective criteria and a prize of a £25 book token or equivalent overseas voucher, will be awarded by Mike Mudge to the best solution arriving by the closing date.

● Stop press! A communication from Roger Tirtia of Waremme, Belgium requests a proof (assuming it to be correct) of the result that: all integers that are not the sum of less than four squares are given by the formula, $2^{2n}(8k+7)$ where n,k = 0,1,2,3… Can anyone help?

● Review of *Numbers Count* 143, March 1995, Beginners Lesson One, (GCD) Wang Tiles and Dr Reg Silvapulle's problem.

The latter problem was solved most elegantly, using Fermat's method of descent and the well known formula for representing a Pythagorean Triple as $2k(r^2+s^2)$, $2k(r^2-s^2)$ & 4krs, by Michael Behrend of Cambridge. Details on request to MM. The introduction of a Beginners Start Here, with an investigation of Greatest Common Divisor of a set of positive integers produced a mixed response. Many found the whole idea too simple to be worthwhile… one respondent returned to the recursive Euclid's Algorithm as he had learnt it at school c.1920 and programmed it in Pascal as a Function.

The Wang Tiling Problem produced responses at many levels; worthy of mention is that of Fred Nooitgedacht of Apeldorn in The Netherlands. His interest began when he met "an old man in 1986, who had been trying to trace all 2,339 ways in which the 12 petominos can fill a 10 x 6 rectangle". He uses "backtracking" and invites mail at Pallietergaarde 219, 7329 HC Apeldorn, The Netherlands. Fred's second mailing solves the problem as posed. The prizewinner is Gustaaf Lahousse, of St Donatuslaan, 4B 1850, Grimbergen, Belgium.

PCW Contributions welcome

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational mathematics, together with suggested subjects and/or specific problems for future Numbers Count articles.