Wondrous Numbers

... a sight to behold! Steer your gaze towards these number nuggets, presented by Mike Mudge.

This topic is believed to have originated with Charles Ashbacher: *J. Recreational Mathematics*, Vol. 24(1) 12-15, 1992. However, readers beware — this article contains a serious misprint which has been self-propagating to the present day.

WONDROUS numbers are members of a series of integers (positive?) which always converges to 1. This series is given by:

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If n be even, the next n = n/2:
If n be odd, the next n = 3n+1
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Hofstadter used the term “Wondrous” as these numbers are so simple and awesome and can be produced in a solitaire-like game. See D.R. Hofstadter, Godel, Escher, Bach: *An Eternal Gold Braid*, Basic Books Inc, New York, pp400-402, 1979. Now B.C. Wiggin, *Journal of Recreational Mathematics*, 20:1, pp52-56, 1988, Wondrous Numbers a Conjecture about the 3n + 1 Family, extended the concept as a genus for the numbers 2 through 12. For example, he defined the (n/3, 4n+1) WONDROUS NUMBERS as:

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Divide n by 3
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If there is no remainder, the quotient is the next n: If the remainder is 2, the next n = 4n - 1: If the remainder is 2, the next n = 4n + 1.

This series also converges to 1 and has been tested at least upto 100000. Now Wiggin extended the concept further to (n/D, (D+1)n + 1) as follows:

Any integer n greater than or equal to (D - 1) may be directed through a series of iterations as a function of n congruent to R modulo D:

If R = 0 then the next n is the quotient n/D: if R = 1(1) (D-2) then the next n is n(D + 1) - R (*NOT* n/(D + 1) - R as in the Ashbacher paper), and finally, if R = D - 1 then the next n is n(D + 1) + 1. This series (more correctly referred to as a sequence — M.M.) ultimately converging to n less than D.

Now, Ashbacher is quoted as originating this topic (M.M. line one) despite the earlier references, on account of his extensive computer investigation... using nine computer programs, one for each of the species three through twelve and searching to 14000000 for each divisor. No infinite series were found, for divisors 3 to 8 all numbers terminated before 1000 cycles, for divisor 9 only 13655938 passed the 1000-cycle mark with 1035 cycles, for divisor 10 there were exactly 1000 cycles for 5646474 and 1037 cycles for 13667102. Ashbacher challenged readers to investigate further...

A challenge recently taken up by Brendan Woods of Dublin, who has failed to get termination with (D,N) = (13,70), (14,75), (58,59), (82,83), (198,199)...

PROBLEM W. Readers are set the task of investigating these generalised WONDROUS NUMBERS, confirming the findings of Ashbacher & Woods, considering further empirical evidence and (hopefully) producing a complete theory of the behaviour of these sequences.

Feedback from readers

The M500 Society publishes *M500* six times a year for Open University students, staff and “friends”, the subscription is £8 per year, the membership secretary is Sue Barrass, 17 Newhall Road, Kirk Sandall, Doncaster DN3 1QQ. Tel 01302 882476.

Harvey Dubner and Harry Nelson announced c.29th August 1995 that they had found SEVEN CONSECUTIVE PRIMES in ARITHMETIC PROGRESSION. Upto 7 computers, believed to be 486/66s, were used over a two-week period to obtain the estimated 52 computer days, the common difference of the A.P. is 210 and the first term has 97 digits.

Simon Jackson of Hackney has expressed an interest in the behaviour of the function ph(p,x) which essentially counts the number of positive integers less than or equal to x which remain when the complete set are depleted in turn by the multiples of all primes less than or equal to prime p. Thus ph(11,25) commences with the list 1...25, removes the even members (multiples of 2), then the multiples of 2 and so on upto the multiples of 11: leaving 1,13,17,19 and 23. Thus ph(11,25) = n(1,13,17,19,23) = 5. Does any reader have practical experience of this function which is believed (S.J.) to be of importance in certain aspects of data compression?

Any investigation of Problem W may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthen, Dyfed SA33 4AQ, tel 01994 231121, to arrive by 1st March 1996. All material received will be judged using suitable subjective criteria, and a prize in the form of a £25 book token or equivalent overseas voucher will be awarded by Mike Mudge to the “best” solution arriving by the closing date.

Review, May 1995

Problem CJ_1 produced a number of complete proofs, that of Roger Tirtia of Belgium including a method for finding all solutions even when a cube is not present. Roger lists the least numbers that can be written in exactly k distinct ways as the sum of two non-zero squares. For k = 29 N is greater than 10^{15} but not difficult to find as there are only five candidates!

Iain M. Davidson of Carlisle examined $a^2 + b^2 = c^n$ but additionally posed questions about general factorisation of

$$A^n + B^n + ... + Z^n,$$

about the solution of

$$X^3 + kY^3 + k^2Z^3 - 3kXYZ = 1,$$

and (possibly more relevant to most *Numbers Count* readers) also requested information on algorithms used to carry out multi-length integer arithmetic. HOW DOES UBASIC DO THAT? Anthony Stobart of Cheltenham and Mr. Kennedy of Rotherham submitted creditworthy material associated with CJ_1... details on request.

David Broughton of the Isle of Wight generated a superb analysis of the “two person game” in CJ_2 finishing with a hand-held programmable calculator to check his manual theory.

After suitable soul-searching, the prizewinner is Iain M. Davidson of 4 Carliol Close, Carlisle CA1 2QP.

PCW Contribution Welcome

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future **Numbers Count** articles.