# High interest

**Mike Mudge** gets to grips with the Hugo Steinhaus problem, and plays around with powers.

Given an initial positive integer, $n_0$, written as

$a_r a_{r-1} \ldots a_1 a_0$ in radix 10;

that is,

$n_0 = a_0 \times 10^0 + a_1 \times 10^1 + \ldots a_{r-1} \times 10^{r-1} + a_r \times 10^r$,

(where the ai are digits from 0,1,2..9) an interactive process is defined by repeatedly forming the sum of the $q^{th}$. powers (where q is a positive integer) of these digits. Thus,

$n_1 = a^q{}_0 + a^q{}_1 + \ldots = a_{r-1}{}^q + a^q{}_r$

etc.

**Case (1) q=2.** It has long been known that this iteration leads either to the cyclic sequence of length one (1), or to the octad (4, 16, 37, 58, 89, 145, 42, 20). See Arthur Porges', *A Set of Eight Numbers, American Mathematical Monthly*, Vol.52, pp 379-382, 1945.

**Case (2) q=3.** Here, there result five possible cyclic sequences of length one (1), (153), (370), (371), (407); two possible cyclic sequences of length two (136,244), (919,1459); together with two such cyclic sequences of length three (55,250,133) and (160,217,352). See Kiyoshi Iseki, *A Problem in Number Theory, Proceedings of the Japanese Academy,* Vol.36, pp 578-583, 1960.

Moving on in the natural way to

**Case (7) q=8.** Ichiro Takada, *Computation of Cyclic Parts of the Steinhaus Problem for Power 8, Mathematics Seminar Notes, Kobe University*, Vol.7, No.3, pp 543-546, 1979 reveals four cyclic parts of length one (1), (24678050), (24678051), and (88593477); one cyclic period of length three (54642372,7973187,77124902); one cyclic period of length twenty-five (9514916,....65602117) and one cyclic period of length one hundred and fifty-four (14889347,....67672102).

Note; these latter two cycles can be entered from $n_0$=2 and $n_0$=3 respectively, whilst the smallest number leading to the cyclic period of length three is 111348.

**Case (9) q=10.** The only cyclic period known to the writer is of length one and is (4679307774) see, for example, Tim Sole, *Computer Bulletin*, page 9, 1981.

**PROBLEM STEINHAUS A.** Investigate the Steinhaus Problems of order 8, 9 and 10 confirming and completing the results quoted above.

**PROBLEM STEINHAUS B.** Generalise the investigation to q=11,12, etc and attempt to construct a theoretical model for predicting the behaviour of this algorithm.

**PROBLEM STEINHAUS C.** Investigate the effect of change of radix of the arithmetic from ten to say 8 and 12, i.e from decimal to octal and hexadecimal.

**PROBLEM ASSOCIATED STEINHAUS.** What happens when the sum of the $q^{th}$. powers of the digits is replaced by the $q^{th}$. power of the sum of the digits?

**UPDATE ON SOPHIE GERMAIN PRIMES.** These numbers were introduced to *Numbers Count* readers in *PCW* December 1993. P is a Sophie Germain Prime if and only if 2P + 1 is also a prime. SG proved a truly beautiful theorem: If P is a Sophie Germain prime, the there are no integers x,y,z different from 0 and not multiples of P, such that $x^P + y^P = z^P$. A recent communication, via Nigel Backhouse, advises that Harvey Dubner has regained the record for the largest SG Prime (at 3rd October, 1995) with the same 5082 digit number

$P = 2687145 * 3003 * 10^{5072} - 1$.

**WHAT ABOUT AN UPDATE ON WILSON PRIMES?**

Readers of *PCW* June 1984 learnt that if P is a prime and N! (factorial n) denotes the continued product of all the integers from 1 to N inclusive, then (P-1)! is congruent to -1, modulo P: which means that the WILSON QUOTIENT,W(P)=((P-1)!+1)/P is an integer. Now if W(P) is congruent to 0,modulo P; which means that (P-1)! is congruent to -1 modulo $P^2$, then P is called a WILSON PRIME. Only three such numbers are known; 5, 13 and 563....causing H.S. Vandiver to observe "This question (whether there are infinitely many Wilson Primes) seems to be of such a character that if I should come to life any time after my death and some mathematician were to tell me it had been definitely settled, I think I would immediately drop dead again."

**CAN POWERS OF TEN BE INTERESTING?**

An INTERESTING POWER OF TEN is defined as one which can be expressed as the product of two ZERO-FREE factors.

$10^{18}$ together with $10^{33}$ = 8589934592 x 116415321826934814453125

are the only interesting powers of ten between $10^9$ and $10^{5000}$. Reference, Mike Mudge, *Computer Weekly*, 5 September 1985. Do there exist infinitely many interesting powers of ten? Is there an algorithm (other than search and test!) for finding them? What are the next terms in the sequence 18,33,....used in this context? Any investigations of the Steinhaus problems and other matters referred to above may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthen, DYFED SA33 4AQ to arrive by 1st April, 1996. The sender of the best solution will receive a £25 book token. Contributors should include details of hardware, coding, run times and results.Please note that material can only be returned if a stamped addressed envelope is provided.

**Review of Numbers Count -146-, *PCW* June 1995**

A disappointing overall response — could this be a function of the summer vacation or the choice of topics? There is a great deal of Smarandache-related material available, the latest collection being *Smarandache Function Journal*, vol. 6, no.1, June 1995, ISSN 1053-4752, published by the Department of Mathematics, University of Craiova, Romania, and available from Dr R. Muller, Number Theory Publishing Co, PO. Box 10163, Glendale, Arizona, 85318-0163, USA. However, the "suitable subjective criteria" has yielded a prizewinner from The Permutation Problem, Mark W. Lewis of 6 Hill Drive, Failand, Bristol BS8 3UX, with a neat combination of computation and algebraic theory. Details available on request.

**PCW Contribution Welcome**

**Mike Mudge** welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future **Numbers Count** articles.