Boxing clever

Should Blind Barpersons wear Boxing Gloves? **Mike Mudge** does some ducking and diving.

The major investigation this week has been suggested by Nigel Hodges of Cheltenham, following upon the work of Richard Ehrenborg and Chris Skinner, *Journal of Combinatorial Theory*, May 1995, pp249-266, and the earlier treatment by Martin Gardner, *Scientific American*, Feb/March 1979.

Statement of Basic Problem. Four glasses are arranged symmetrically on a circular tray. The barperson selects two positions, the tray is then rotated by his opponent. The barperson can then touch the glasses which have just occupied the chosen positions and is permitted to invert zero, one or both of them. The process is repeated and the object is to end up with all the glasses pointing the same way.

Now this can always be achieved in five moves, as shown in *Fig 1*.

Fig 1 Five moves for the Barperson Problem

```
1) Select either diagonal and turn both glasses up.             u      •
                                                                •      u
2) Select any side and turn both glasses up.                    u      u
                                                                •      u
   Given no win, the configuration is given by:                 u      u
                                                                d      u
3) Select a diagonal. If it displays u-d invert
  d to win, otherwise invert one of the u to yield:             u      u
                                                                d      d
4) Select a side. If it is u-u or d-d invert
   both to win, otherwise invert both to yield:                 d      u
                                                                u      d
5) Invert either diagonal to win.
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Extension to the Basic Problem

Generalise the number of glasses from four to n and ask:

(i) What is the minimum number of "hands" needed by the barperson?

(ii) What is the maximum number of moves needed to guarantee a win?

Now to the boxing gloves... In this extension, the barperson is not only blind but unable to tell which way up the glasses are! It is claimed that the four-glass problem can still be solved in seven moves; while the problem of 2^n glasses can be solved in $(2^{2^n-1})-1$ moves with the requirement that the barperson has at least 2^{n-1} hands and further that this solution is optimal.

Problem NH. Simulate the BLIND BARPERSON PROBLEM, with/without boxing gloves; either in an interactive mode or with built-in (random) choice, and collect data for various numbers of glasses and various configurations of barperson (i.e. number of hands variable). Using the result of such simulation or otherwise, solve the n glass problem.

Note: The theoretical results appear to depend upon sophisticated algebra of finite groups.

A Quest from Teck-Sing Tie in Sarawak:

Problem TST (I). Is the general factorisation problem an NP-complete problem? i.e. Has factorisation of an arbitrarily large composite number been proven to be equivalent to an NP-complete problem?

Problem YSY (II). Can we prove that the difference between a prime, p_n, and the next prime, p_{n+1}, can never exceed $((\ln(p_n))\ln(\ln(p_n)))^2$?

Further, can we prove that there is at least one prime between p_n and p_n plus the above logarithmic expression, and that this is the smallest possible gap? Some empirical investigation *may* disprove this!

Problem MH. José Castillo has conjectured that the (Smarandache Expression) $x^y + y^x$ where x,y are co-prime integers greater than or equal to 2 generates only a finite number of prime values.

Mario Hernandez at Univxiq (whatever that is?) would like an analysis of prime occurrences. *viz*

$3^2 + 2^3 = 17$

$4^5 + 5^4 = 1649 = 17.97$

News Flash. At October 9th 1995 Harvey Dubner used his Dubner Cruncher (details from Harvey Dubner <951017164327 70372.1170 JHD102-1@CompuServe.COM>) to find after a ONE DAY search a record twin prime pair:

$P = 190116*3003*10^{5120}$

and

$Q = P + 2$

each has 5129 digits.

Any investigations of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthen, Dyfed SA33 4AQ, tel 01994 231121, to arrive by 1st May 1996. All material received will be judged using suitable subjective criteria and a prize in the form of a £25 book token or equivalent overseas voucher will be awarded, by Mike Mudge, to the "best" solution arriving by the closing date. Such contributions should contain a brief description of the hardware used, details of coding, run times and a summary of results obtained. Additionally, any comments upon the specific problem areas covered this month, together with any references to any published, or unpublished, work in these areas, would be greatly appreciated.

Please note that material can only be returned if a suitable stamped addressed envelope is provided.

Prizewinner, Numbers Count -147- July 1995

This month's prize goes to George Sassoon, of Ben Buie Lodge, Lochbuie, Isle of Mull, Argyll PA62 6AA, for his efforts in the factorisation of large numbers and in inspiring co-operation between computer users to attack (in an effective manner) a major task.

PCW Contributions welcome

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future **Numbers Count** articles.