Chiefs and **Indians**

An Indian tribal leadership election problem together with a Portuguese string comparison algorithm. Presented by **Mike Mudge**.

PROBLEM AB

Suggested by Alan Bleeze of West Sussex

The investigation begins with a fairly simple puzzle: the current chief of a tribe of 50 Indians decides who is to be chief next year by forming a circle with more than half of his tribe. He then starts counting around the circle, from the person next to him, every third person dropping out. The single Indian left at the end is the new chief. In the event, the lucky one is (surprise!) the old chief. Question: how many were there in the initial circle?

The easiest way to solve this is to work it backwards: i.e. start with a degenerate circle consisting of just the chief and add an Indian at each count of three. This gives possible answers of 2,13,20,46,157,.., only 46 lying (as required) between 26 and 51. This gives rise to the question: what series of possible answers are available for different counting values?

Alan used ANSI C to display a series up to 5000 for n = 1 to 10. Artificially inserting 1 at the start of every series... if the chief starts alone he must be the last one left. n = 1 clearly yields all positive integers. n = 2 seems to yield $2^x - 1$, while for example n = 8 yields

1,3,13,15,26,1276,1905,2844....

Is there an algorithm to produce the series directly without going repeatedly around the circle?

Alan has an alternative approach: evaluate the series for limit, say 50, and for n=1 to, say, 100 and display the results on a square grid. This appears to produce a recognisable pattern for which values of n the numbers 1,2,3,4 appear in the series, enabling one to predict a series for any value of n up to a limit of 4; but for numbers greater than 4 the pattern is less obvious.

PROBLEM AY

Suggested by A. Yassine of Lisbon, Portugal

System in use AMD 386DX 40MHz

Programming Languages

Turbo Pascal 6, Assembler

The Problem For the sake of simplicity, suppose that we have the following:

```
TYPE ARRAY A = 1 to 9 of 1 to 50
TYPE ARRAY B = 1 to 3 of A.
B:=((1,2,3,4,5,6,7,8,10),(1,2,3,4,
5,6,7,8,20),(1,2,3,4,5,6,7,8,50))
```

Notice that in each array elements are sorted into ascending order and furthermore there is no duplication within the array. It is required to compare the first array with the other two and to conclude that they differ by only one element. The value or position of this element in either array is not required.

The trivial approach is to compare each element of the first array with each element of each of the others. Mr. Yassine finds some drawbacks to this approach: in this simple example 162 comparisons are needed, and in his real problem the arrays are very large and such a comparison routine written in Assembler takes a prohibitively long time. He requests an algorithm that is short enough to allow comparisons between sub-arrays without the necessity of comparing each element. He asks, "Is there any mathematical process that permits the use of SUM, DIFFERENCE, SQRT, etc. as a basis for a comparison?"

Alan Cox has produced two alternative approaches which deal with the simple problem in 81 and 45 comparisons. Is this latter "best-possible"? If so, how does the figure of 45 increase with the number of sub-arrays and the dimension of each?

STOP PRESS. Alan Cox 5/1/96 Christmas Quiz by Adrian Berry, *Sunday Telegraph* 23/12/95, offered £450 for the first answer to the following: "Given that 3,5;5,7;11,13; and 17,19; are consecutive prime pairs, that is they are prime numbers separated only by two and which do not have any other primes between them. "What is the first group of FIVE consecutive prime pairs?"

Alan was initially unhappy with the double occurence of 5 in the above list and so proceeded to

9419,9421;9431,9433;9437,9439; and 9461,9463

Using UBASIC, in particular its nxtprm(n) function, Berry's problem was readily solved. However, on a 286 machine the natural extension to SIX consecutive twin primes produced no results upto about 169 million. (4×10^9 is available from UBASIC.) Alan asks for information on this sequence and "perhaps more interestingly, for an estimate of the chance of such a sequence occurring (as a function of n) making use of the known distribution of primes."

Any investigations of problems AB & AY above together with comments on n-tuples of prime pairs may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthen, Dyfed SA33 4AQ, tel 01994 231121, to arrive by 1st July 1996. All material received will be judged using suitable subjective criteria and a prize in the form of a £25 book token or equivalent overseas voucher will be awarded, by Mike Mudge, to the "best" solution arriving by the closing date.

Review of Numbers Count, September 95

"Pretty poly! and nested bubbles"

These topics produced a considerable response. Numerous responses on the nested bubbles referred to Catalan Numbers and suggested the use of J. Riordan, *An Introduction to Combinatorial Analysis* and also F. Harary, *Graph Theory*; also work by John Gilder, *Mathematics in Schools*, March 1987 and September 1987. The prize-winning entry by Jon McLoone of 10 Blenheim Office Park, Lower Road, Long Hanborough, Oxfordshire OX8 8LN, actually draws the 1842 alternatives for 10 bubbles using Mathematica version 2.2 on a Macintosh Quadra 600 in 50 seconds.

PCW *Contributions welcome*

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles.