Spring into **action**

Mike Mudge presents a spring-time miscellany — hailstones, factorials and points on a graph.

HAILSTONES (PCW January 1992) / WONDROUS NUMBERS (PCW December 1995)

... an alternative approach

Nigel Hodges of Cheltenham has studied the above problems extensively and has reduced the special case "3n + 1" to the following:

Problem NH. Are there any powers of 2, other than 2**3, which satisfy 6**n less than 2**n less than 6**n + 3**n**? More generally, Nigel asks if, given real a & b greater than 1 (integers if readers prefer) how many solutions can be found which satisfy a**n less than b**n less than a**n + d where d can either be a constant value or can increase with n. (But must be smaller than a**n.) In particular, are the solution sets finite or infinite?

** Anyone finding a second solution has cracked the "3n+1" problem, details of the link from Nigel via M.M., and is assured of instant fame!

Some Diophantine Equations involving the Factorial Function

Recall that given any positive integer n, the factorial of n is defined by n! = 1x2x3x..xn, while 0! = 1 for convenience, further Diophantine Equations are those for whch solutions are sought in integers only. The following problems supplement those of *PCW* July 1989 where it was explained that ERDÖS had examined the equation $n! = 2^a \pm 2^b$ finding solutions for n = 1,2,3,4 & 5 only.

Now, as early as 1937 Erdös and Oblath considered the equations

$$n! = x^p \pm y^p$$

and also

$$n! + m! = x^p$$

for positive integers m,n,p,x & y. Their listing appears to consist of (the publication is in German!)

$2! + 2! = 2^2$, $3! + 2! = 2^3$, $3! - 2! = 2^2$, $5! + 4! = 12^2$ also $2! = 1^p + 1^p$ and $12^2 + 24^2$.

Problem EO. Investigate the above equations, together with the associated problem when the two powers in the first equation may not be equal. Obtain empirical data to test my interpretation of "Uber diophantische Gleichungen der Form etc." (usw.) and, if possible, make some theoretical analysis of the results.

A Spring Challenge

Problem MM. In general y = f(x) can be represented by a graph on which there may be points with integer, rational (fractional) or irrational co-ordinates, e.g. y = 3x + 1 is represented by a straight line, here (1,4) is an integer point, ($^1/_3$,2) is partly integer and partly fractional while ($3^{1/2}$, $3^{3/2}$+1) is irrational.

Consider the function

$$y^2 = x(x^2 + p)$$

where p is a PRIME NUMBER. For a given p determine fractional points on this curve i.e. points with fractional co-ordinates, e.g. p=5 leads to x = 1/4, y = 9/8 while p = 13 leads to x = 9/4 and y = 51/8. Determine a general method for the construction of such points, illustrate graphically. (Consider the result of changing the powers/sign present in the equation above.)

FEEDBACK

Perfect Digital Invariants (PDI's)

These are integers which are equal to the sum of the n^{th} powers of their individual digits in a given basis. Following upon the Steinhaus Problem, PCW January 1996, Henry Ibstedt has sent a quotation from Madachy's book *Mathematical Recreations* (Dover, 1979) page 164: "It is a marvel that a tenth-order PDI should have been discovered: 4,679,307,774... There must be many other such digital invariants of higher order, but the numbers get larger and more difficult to work with. Recently the work has been extended to the seventeenth order."

Henry also refers to an article in the *Journal of Recreational Mathematics* (latest issue) "Variants on Perfect Digital Invariants" but presents his own result — a 21-digit PDI or order 21 128...252.. asking is this the present record?

● **Mersenne Primes**

These are primes of the form $2^n - 1$. Eric Adler has been beta-testing the new version of Maple (Maple 4 The Power Edition) and has found the 31st and 32nd Mersenne Primes in less than twenty minutes on his 16Mb 100MHz Pentium. These are

$2^{216091} - 1$ and

$2^{756839} - 1$ (227832 digits).

He followed these with

$M_{33} = 2^{859433} - 1$

having 258716 decimal digits. This exercise is indicative of the evolution of the Personal Computer, as the final calculation was carried out on a supercomputer — the Cray at Harwell in about January 1994, I believe.

● **Some other large primes**

Factorial Primes: 3610! - 1 and 30507! - 1 having 11277 & 10912 digits respectively; C. Caldwell 1992/3.

Primorial Primes: Define n* as the product of all primes less than or equal to n, then we have 24029* + 1 and 23801* + 1 with 10387 and 10273 decimal digits respectively; C. Caldwell 1993. These are the largest primes of each type published... Unless any reader knows better?

Any investigations of the above problems may be set to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthen, Dyfed SA33 4AQ, tel 01994 231121, to arrive by 1st August 1996.

All material received will be judged using suitable subjective criteria and a prize in the form of a £25 book token or equivalent overseas voucher will be awarded, by Mike Mudge, to the "best" solution arriving by the closing date.

Smarandache slip-up

The Enigma of Smarandache continues: is it worthwhile? Response to Numbers Count -150- October 1996 did not warrant the award of a prize. First International Conference on Smarandache Type Notions in Number Theory, August 21-24 1997. Information from C Dumitrescu, Mathematics Department, University of Craiova, Romania. Tel (40) 51-125302; Fax (40) 51-413728 (for Dumitrescu); email Research37@aol.com

PCW *Contributions welcome*

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles.