Sequence of events

Descriptive Number Sequences, presented by **Mike Mudge**.

THIS APPARENTLY NEW AND certainly fascinating topic has been suggested by Jonathan Ayres of Leeds. The sequences are denoted by $ds_n(m)$ where n is the index of the sequence and m is the original number. There is a simplified version of the GLEICHNISZHLEN-RIEHE sequence with the property that the next number in the sequence describes the number of each digit in the previous number.

So, taking the case of ds(0) in *Fig 1*.

Fig 1

Gleichniszhlen-Riehe sequence for ds(0)

$ds_1(0) = 10$ (1 zero in previous number, 1 is the digit number and 0 is the occurrence number)
$ds_2(0) = 1011$ (1 zero and 1 one in previous number)
$ds_3(0) = 1031$ (1 zero and 3 ones in previous number)
$ds_4(0) = 102113$ (NB. Because there are no twos in previous number the 0 twos are not listed, so $ds_4(0)=102113$ instead of 10210213. (I will deal with this case later.)
$ds_5(0) = 10311213$
$ds_6(0) = 10411223$
$ds_7(0) = 1031221314$
$ds_8(0) = 1041222314$
$ds_9(0) = 1031321324$
$ds_{10}(0) = 1031223314$
$ds_{11}(0) = 1031223314$, and so on

After ds11(0) all further numbers in the sequence are equal to 1031223314. This is a *self-descriptive number*, i.e. it describes itself. For example, 1031223314 is composed of 1 zero, 3 ones, 2 twos, 3 threes and 1 four = 1031223314.

From my investigations the self-descriptive numbers are:

22
10311233
21322314, 21322315, 21322316, 21322317, 21322318, 21322319
31123314, 31123315, 31123316, 31123317, 31123318, 31123319 *
1031223314, 1031223315, 1031223316, 1031223317, 1031223318, 1031223319 *
3122331415, 3122331416, 3122331417, 3122331418, 3122331419 *

The asterisked lines are related families because the final 1n is not important as n is not involved with the rest of the number.

This leads to my first question:

(1) Is this an exhaustive list of self-descriptive numbers?

Sequences which do not lead to self-descriptive numbers instead lead to *amicable descriptive sequences*. For example, in the case of ds(4), see *Fig 2*.

$ds_{10}(40) = ds_{12}(40)$, so this sequence has entered into a *recurring sequence* of numbers with a *period* of 2, because $ds_n(40) = ds_{n+2}(40)$, $n \Rightarrow 10$.

104122232415 and 103142132415

Fig 2

Amicable descriptive sequences for ds(40)

$ds_1(40) = 1014$
$ds_2(40) = 103114$
$ds_3(40) = 10311214$
$ds_4(40) = 1041121314$
$ds_5(40) = 1051121324$
$ds_6(40) = 104122131415$
$ds_7(40) = 105122132415$
$ds_8(40) = 104132131425$
$ds_9(40) = 104122232415$
$ds_{10}(40) = 103142132415$
$ds_{11}(40) = 104122232415$
$ds_{12}(40) = 103142132415$, and so on

are known as an amicable descriptive pair of numbers, because

$ds_1(104122232415) = 103142132415$
and $ds_1(103142132415) = 104122232415$

There are also amicable descriptive triplets such as

10414213142516 - 10512213341516 - 10412223142516

which have a period of 3. The amicable descriptive sequences are shown in *Fig 3*.

From this I define ds(x) to be the lowest recurring value of dsn(x), so that ds(x) is either a self-descriptive number or ds(x) is the lowest member of an amicable sequence, i.e. ds(0) = 1031223314.

Fig 3

Amicable descriptive sequences for triplets

Period 2

103142132415 - 104122232415

314213241516 - 412223241516,
314213241517 - 412223241517,
314213241518 - 412223241518,
314213241519 - 412223241519

41421314251617 - 51221334151617,
41421314251618 - 51221334151618,
41421314251619 - 51221334151619

1051421314152617 - 1061221324251617,
1051421314152618 - 1061221324251618,
1051421314152619 - 1061221324251619

5142131415261718 - 6122132425161718,
5142131415261719 - 6122132425161719
106142131415162718 - 107122132415261718,
106142131415162719 - 107122132415261719,
614213141516271819 - 712213241526171819,
10714213141516172819 - 10812213241516271819

Period 3

10414213142516 - 10512213341516 - 10512223142516
10414213142517 - 10512213341517 - 10512223142517
10414213142518 - 10512213341518 - 10512223142518
10414213142519 - 10512213341519 - 10512223142519
41421314251617 - 51221334151617 - 51222314251617
41421314251618 - 51221334151618 - 51222314251619
41421314251619 - 51221334151619 - 51222314251619

(2) Is this a complete list of the amicable descriptive sequences?
(3) Are there any of higher period?

Any investigations of these three questions may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthenshire SA334 AQ, tel 01994 231121, to arrive by 1st September 1996. All material received will be judged using suitable subjective criteria and a prize will be awarded to the "best" response arriving by the closing date.

PCW *Contributions welcome*

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles.