Back in **sequence**

Descriptive Number Sequences, part two, presented by **Mike Mudge**.

Continuing the study of Numbers Count, June 1996: Recall the definition, due to Jonathan Ayres, of Leeds: $ds_n(m)$ where n is the index of the sequence and m the original number. Thus:

$ds_1(0) = 10$

because the original number consists of 1 zero; whilst

$ds_2(0) = 1011$

because $ds_1(0)$ consists of 1 zero and 1 one.

Problem A. Is there a way of deciding if a given initial number, x say, leads to a self-descriptive number (such as 1031223314) without calculating the whole descriptive sequence?

Empirical evidence suggests that as x increases, the likelihood of a sequence becoming self-descriptive decreases. Why is this?

Problem B. Is there any function which relates the chances of a number becoming self-descriptive with the magnitude of the number?

COMPLETELY DESCRIPTIVE SEQUENCES, Ds(n)

These are similar to descriptive sequences, but the next number in the sequence refers to all the digits zero to nine i.e. it does not omit the reference to non-occurring digits.

$Ds_1(0) = 10010203040506070809$,
$Ds_2(0) = 10021121314151617181 9$

This process converges to the amicable descriptive pair:

$Ds_6(0) = 10714213141516171819$,
$Ds_7(0) = 10812213241516271819$

Problem C. Do all numbers n lead to the above amicable descriptive pair?

WHAT HAPPENS IN DIFFERENT NUMBER BASES?

In binary, for example,

$ds_1(0) = 10$, $ds_2(0) = 1011$ whilst
$ds_3(0) = 10111$

We have 11 ones since 3 is represented in binary as 11; subsequently

$ds_9(0) = ds_{10}(0) = \ldots = 1101001$

a self-descriptive number in binary, having three zeros and four ones.

Example: in base 6 there is an amicable descriptive pair consisting of

103142132415 & 104122232415

Problem D. Are there any number bases with period four or larger amicable descriptive sequences?

DESCRIPTIVE SEQUENCES OF ORDER GREATER THAN ONE

Here the digits are regarded in groups of order n which may be either CONSECUTIVE...TYPE I, or GROUPED..TYPE II?

In type 1 the number zero generates the following:

$dsc^2_1(0) = 0100$

which is then split into 01, 10 & 00

$dsc^2_2(0) = 010001010110$ and $dsc^2_3(0) = 0200040104100111 \ldots$

whilst in type II the number zero generates the following:

$dsg^2_1(0) = 0100$

(because order two uses two digits so 0 goes to 00 and 1 goes to 01)

$dsg^2_2(0) = 01000101$

i.e. one zero and one one.

$dsg^2_3(0) = 01000301$

i.e. one zero and three ones etc.

It is found that $dsg^2_{989}(0)$ and its amicable descriptive partner are each 395 digits long; whilst $dsc^2_{41}(0)$ having 395 digits also is part of an amicable descriptive pair.

Problem E. Analyse completely the behaviour of type I & type II descriptive sequences of order two, consider the extension to higher orders. (Remember the order is the size of the subsets of digits being counted.)

TWO-DIMENSIONAL DESCRIPTIVE SEQUENCES, $^2DS(n)$

Descriptive sequences can be generalised from one-dimensional "lines" of numbers to two-dimensional "planes" of numbers. One way to do this consistently is to define the columns, m, of $^2DS_{n+1}(x)$ to be equal to ds_1(row m) as is illustrated by the following example:

$^2DS_1(0) = 1_0$ (because $ds_1(0) = 10$)
thus
$^2DS_2(0) = 11_{10}$ (because $ds_1(1) = 11$ and $ds_1(0) = 10$) repeated iteration leads to:

$$^2DS_5(0) = \begin{matrix} 4 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 \\ & 3 & & 1 \\ & 1 & & 2 \end{matrix}$$

Jonathan Ayres has failed to discover any two-dimensional self-descriptive or amicable descriptive sequences, having investigated up to $^2DS_{1000}(0)$ and beyond. However, he observes that $^2DS(\ )$ must lead to a recurrent sequence because it is fixed in size. The biggest $^2DS(\ )$ gets in size is 19 rows by 19 columns and since each position contains a digit 0..9 then there are 10^{361} possible values for $^2DS(\ )$, but half the possible positions on average are spaces and half the remaining numbers are fixed because they are the digit number, so maximum period is about 10^{81}.

Problem F. Investigate two-dimensional descriptive sequences with a view to finding self-descriptive or amicable descriptive patterns.

Any investigations of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, Carmarthenshire SA33 4AQ, tel 01994 231121, to arrive by 1st October 1996. All material received will be judged using suitable subjective criteria and a prize will be awarded by Mike Mudge, to the "best" entry arriving by the closing date.

Feedback: November 1995 — Squambling

This proved to be a remarkably popular topic. Why? Gareth Suggett established the answer to the original *Sunday Times* problem as 46, for which one iteration of the squambling function gives 232, and a second gives 47. He found all of George Sassoon's loops and lists a 105-step loop, 40372656... whose smallest entry is 5 and largest entry is 43055027. He found modsquam less interesting, being monotonic decreasing and ending (always) with 1. Nigel Hodges proved that squambling sequences and their various generalisations cannot diverge. However, this month the prize is awarded to G.D. Williams of 18 Mawnog Fach, Bala, Gwynedd LL23 7YY, who displays an awareness of the problems of integer overflow even when programming in Turbo C++. Mr Williams has noted the basic difference in the behaviour (as he perceives it) between sqm( ) & modsqm( ).

There is scope for further investigation of this function, in particular when the number base is different from ten.

PCW Contributions Welcome

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future *Numbers Count* articles.