Golomb rules, **OK**

Mike Mudge deals with the concept of perfect and imperfect rulers.

"Golomb rulers"

…with a pre-metric introduction

(Inspiration acknowledged from Malcolm E. Lines' Think of a Number *[1990. Adam Hilger. ISBN 0-85274-183-9] chapter 11, pp 101-105.)*

Consider a one-foot ruler having only inch markings: for the benefit of younger readers this represents a uniform scale of length, with 12 equal sub-divisions.

We see that there are 12 ways of measuring one unit of length, *viz.* 0-1, 1-2, 2-3,…11-12; also, seven ways of measuring six units of length, *viz.* 0-6, 1-7,…6-12; with four ways of measuring nine units of length 0-9, 1-10, 2-11, 3-12. Clearly,there is considerable redundancy in this instrument.

Starting with a "trivial prototype" ruler of unit length marked 0 & 1, this measures one possible distance in one possible way. However, a two-length ruler with three markings at 0, 1 & 2 has already introduced an inefficiency since it measures one unit in two different ways, *viz.* 0-1 & 1-2. However, if the marks are at 0, 1 & 3 we have a ruler measuring distances 1, 2 & 3 each one way only.

Pencil and paper study should convince the reader that it is not possible to construct a ruler which will achieve this for either 4 or 5. Marks at 0, 1, 4 & 6 generate such a ruler (called PERFECT) of length 6, since either of the distances 1, 2, 3, 4 & 5 can be measured in one way only. This idea is originally due to Professor Solomon Golomb of the university of Southern California; see later reference.

Now a ruler with five marks can measure ten distances, therefore if it were a PERFECT ruler it would be of length 10. Note that a ruler can be IMPERFECT in two distinct ways:

(a) there may be some distances which cannot be measured;

(b) there may be some distances which can be measured in two or more ways.

The "next best thing" to the non-existent perfect five-mark ruler might possibly be defined as one that contains each measurable distance only once, but which is unable to measure every possible distance up to the length of the ruler. Clearly not an adequate definition since a five-mark ruler, marked at 0, 4, 10, 27 & 101, measures distances of 4, 6, 10, 17, 23, 27, 74, 91, 97 & 101 each one way only. the challenge is to find the SHORTEST ruler which does not measure any one distance in more than one way.

The shortest five-mark ruler is of length 11; with mark positions at 0, 1, 4, 9, 11, i.e. one unit longer than the PERFECT TEN. The only length less than ten which it cannot measure is 6. In general, the shortest ruler with n-marks is called the "n-mark Golomb ruler" in honour of its inventor. Malcolm Lines lists all of the Golomb rulers known to him in *Fig 1.*

Fig 1 Golomb rulers

Number of marks	2	3	4	5	6	7…	13	14	15
Golomb length	1	3	6	11	17	25…	106	127	151

The fifteen-mark Golomb ruler has marks at 0, 6, 7, 15, 28, 40, 51, 75, 89, 92, 94, 121, 131, 147, 151.

Now to research! There exists a formula which yields the shortest length that a Golomb ruler with any particular length can possibly have. This yields the entries in row L (Lower bound) shown in *Fig 2.*

Fig 2 The entries in row L

Number of marks	16	17	18	19	20	21	22	23	24
Shortest known	179	199	216	246	283	333	358	372	425
L	154	177	201	227	254	283	314	346	380

Problem: Golomb

The following quotation is intended to inspire readers to investigate the problem of Golomb rulers: "In co-operation with a personal computer, it is quite likely that the enthusiast can improve on some of the 'shortest known' rulers in the above table, although a demonstration that the actual Golomb ruler has been located is probably beyond all but the most powerful of today's computers." (*M.E.L. 1990*).

Investigate the table at *Fig 2*, extending where possible and making a serious attempt to quantify the difference between any "shortest known" results, i.e. from a particular algorithm, and the actual length of the Golomb ruler. Further, how do these values differ from **L**?

Any investigation of the above problem, together with comments on the concept of Golomb rulers (which do have applications in both radio astronomy and satellite communications) may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, Carmarthenshire SA33 4AQ (tel 01994 231121) to arrive by 1st November. All material received will be judged using suitable subjective criteria and a prize will be awarded, by Mike Mudge, to the "best " entry arriving by the closing date. Such contributions should contain brief descriptions of the hardware and coding used, together with run times and a summary of the results obtained. (SAE for entries to be returned, please.)

Some simply-posed problems for beginners

P1) Does there exist a positive integer n greater than 7 for which n! + 1 is the square of the integer? It is known that if n exists it must be greater than 1020.
M.Kraitchik (Paris, 1924).

P2) Obtain all solutions in integers of the equation $x3 - y^2 = 18$. It has been proved that the number of solutions is finite but it is not known how many there are.

P3) Do there exist three rational numbers (i.e. fractions with integer numerators and integer denominators) whose sum and product are each equal to 1?

PCW ***Contributions Welcome***

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future *Numbers Count* articles.