Prime candidate

Prime functions take centre stage and hailstones are a big hit, in this month's maths musings. With Mike Mudge.

The first area of investigation this month is due to Jonathon Ayres of Leeds, who writes as follows:

Highest prime function

I have recently been investigating what I have called the highest prime function — HPF(X), which is defined as the highest prime factor of x, so that HPF(7) = 7 and HPF(10) = 5.

1) Highest prime function sequence

HPFS(x_0,x_1) is defined so that

$X_n = HPF(X_{n-1}*X_{n-2}+1)$

So,

```
HPFS(1,2) = 3, 7, 11, 13, 3, 5,
2, 11, 23, 127, 487, 1237,
331,127,21019, 1811, 140983,
2239651, 10005473, ..
```

and

```
HPFS(3,2) = 7,. 5, 3, 2, 7, 5,
3, 2, and so on
```

(this has period 4).

Questions

1). Do all HPFSs eventually lead to recurring sequences? For example, HPFS(x,y) leads to a,b,c,d ... a,b,c,d, and so on. If not, do all the non-recurring HPFS go through all possible numbers? (The function

$HPFS(x_n) = HPF(x_{n-1}*x_{n-2}*... x_1 , x_0 +1)$, starting 2,3,7, 43, 139 ..

has been shown not to repeat, nor is it ever equal to

`5,11,13,17.)`

2). For recurring HPFS, what are the smallest numbers a,b so that HPFS(a,b) has period n, and are there any values of n so there are no HPFS(a,b) with period n?

3). How many different HPFS do numbers converge to? For instance, HPFS(2,3) and HPFS(2,11) converges to the same sequence?

4). What happens for related sequences such as:

a) Lowest prime factor sequence

`LPFS(3,7) = 2,3,1 .....`

b) Highest alliott divisor sequence (not including the number itself)

`HADS(3,5) =8, 1, 3, 2, 1,...`

c) HPFSm(a,b) = HPF(a*b+m)

d) HPFS(a,b,c) = HPF(a*b*c+1)

Highest Prime Hailstone Function

This is similar to the "Hailstone Function", which is defined as: if n is even, then n is divided by 2, else it is multiplied by 3 and one is added.

The highest prime hailstone function,

$HPHF_{(a,b)}(x_n) = HPF(a*x_{n-1}+b)$

$HPHF_{(2,1)}(1)$ = 3, 7, 5, 11, 23, 47, 19, 13, 3, 11, 23, 47, 19, 13, ...

has period 7, with the lowest value in the periodic sequence being 3 and the highest 47.

The table above shows the period, lowest and highest value for the recurring sequence which $HPHF_{(n,1)}$ (1) leads to.

Values for recurring sequence

N	Period	Lowest value	Highest value
2	7	3	47
3	5	2	17
4	7	5	71
5	3	2	11
6	18	13	13219
7	3	2	23
8	12	11	1097
9	5	13	97
10	6	43	15971
11	2	17	47

Questions

1). Do all $HPHF_{(a,1)}$ (1) lead to recurring sequences, and how does the period, lowest and highest value change for different a (for example, a being prime, then the period seems to be quite low)?

2). Do all $HPHF_{(a,b)}$ (x), for fixed a,b and variable x, lead to recurring sequences?

For fixed a and b, is there more than one recurring sequence? For example, $HPFH_{(9,1)}$(1) leads to a sequence with lowest value 13, highest 97 and length 5; and $HPF_{(9,1)}$(41) leads to a sequence with lowest value 37, highest 269 and length 7. If so, how many different recurring sequences?

For fixed a and b, what value of x takes the longest/shortest time to reach a repeating sequence, and what value of x reaches the highest values?

3). Do all HPHF lead to recurring sequences?

Any responses to these problems to be sent to: Mike Mudge, 22 Gors Fach, Pwll-Trap, Carmarthenshire SA33 4AQ (01994 231121), by 1st January 1997.

Spot the difference

Stephen Saxon, of Stockport, has suggested an interesting problem — it combines an area of mathematics predating computers "as we know them" by several centuries, with current programming techniques. The question is, how to fit a polynomial of the lowest possible degree to a set of equally spaced data points? An answer will be provided next month by The Calculus of Finite Differences or, as Stephen calls it, The Newtonian Difference Method.

• PCW Contributions Welcome

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles.