Close **relations**

Mike Mudge presents the relationship between Archimedean Polyhedra and the Tribonacci Series. Mersenne Non-Primes get some attention, too.

In a recent letter John Sharp, of Watford, wrote: "*It is well known that the dodecahedron and the icosahedron are intimately bound up with the Golden Section, which is in turn related to the Fibonacci Series.*" Readers whose knowledge of geometry is minimal may already feel discouraged. However, this is an arithmetic problem.

John has studied the Archimedean Polyhedra known as the snub-cube and the snub-dodecahedron and has found a similar relationship between the former and the Tribonacci Series, defined by the recurrence relationship:

$T_{k+1} = T_k + T_{k-1} + T_{k-2}$

with suitable initial values for

T_0, T_1 & T_2

The constant associated with this series is

t = 1.83928675521416..

this being the positive root of the quartic equation:

$t^4 - 2t^3 + 1 = 0$*

Now, relative to a snub-cube with unit sides, the diagonals have (approximate) length:

A= 1.68501832488972
B = 1.83928675521416
C = 2.16300104263277
D = 2.320124084592509
E = 2.382975767906236
F = 2.434474230834721
G = 2.584293619236854
H = 2.601144274317068
I = 2.657357374421356

$A = (t + 1)^{1/2}$, $B = t$, $E^* = t + 1/t = (2t + 2)^{1/2}$

Intuition tells John Sharp that the lengths of the other diagonals have "*some relatively simple relationships to t*" but how can these be found computationally?

For the snub-dodecahedron with constant

m = 1.943151259243865

there are 28 lengths commencing with:

A = 1.715561499697342
B = m
C = 2.343373277136706
D = 2.467232466141474
E = 2.528610449446665
AA = 4.260575577706465
F = 2.775836816301074
G = 2.782298391314399
H = 3.059283956591891
I = 3.118888631147017
J = 3.144084782738732 down to
BB = 4.294380888587396

There is a database available on the internet called the Inverse Symbolic Calculator (ISC) by J. A. Sloane and S. Plouffe, having, on August 1996, 45 million entries which (reference: "A question of numbers", by Brian Hayes, Scientific American, vol. 84, Jan-Feb 1996) Plouffe foresees expanding to a billion entries. The internet address is www.cecm.sfu.ca/projects/IS/ISCmain.html. Here, the Tribonacci constant is easily found but there is no entry "close to" m. Help!

Mersenne NON-PRIMES

On 3rd September 1996, Cray Research announced that Slowinski and Gage had found the 34th Mersenne Prime, being $2^{1257787} - 1$ with 378632 decimal digits. (*Note: This may not be the 34th in order of magnitude as the search algorithm is not exhaustive*). However, Jonathan Ayres of Leeds, one of our regular readers, drew my attention to certain problems related to Mersenne NON-PRIMES.

Revision note: a Pseudo-Prime to base b is a number, n, such that bn-1-1 is divisible by n. For example, 15 is the smallest pseudo-prime to base 4, because 414 - 1 = 268435455 is divisible by 15. Similarly, 217 is the second smallest pseudo-prime to base 4. 91 is the smallest pseudo-prime to base 3, 341 and 641 are the first two pseudo-primes to base 2, while 161038 is the smallest *even* pseudo-prime to base 2.

A Carmichael Number (or Absolute Pseudo-Prime) is a pseudo-prime to any base. So, 561, 1729, 2821, 1105, 1729, 2464, 2821 are examples of such numbers, $a^{560} - 1$ being divisible by 561 whatever the value of a.

■ **PROBLEMS MNP**

1. Are all non-prime Mersenne numbers pseudo-prime to some base b, and more generally pseudo-prime to some base 2^p? Are there some Mersenne numbers that are Carmichael numbers?

2. Are all non-prime Mersenne numbers pseudo-primes to some base b, where b is not a power of 2, and how does this number relate to p? Furthermore, is there some base b, that is not a power of 2 but is a pseudo-prime basis for more than one Mersenne number?

3. Are all composite xy ± 1 pseudo-primes for some base b, and are there any Carmichael numbers of this form?

Some of the early numerical results relating to problems MNP can be obtained by sending a stamped addressed envelope to Mike Mudge.

Any investigations of Problems MNP and/or advice for John Sharp may be sent to me, Mike Mudge at the address shown in the *PCW* panel here, to arrive no later than 1st March 1997.

INTEGRAL BASES and Computer Experiments due to Shen Lin

I have a further item which follows on from last month's theme, based on an article by P. Shiu. Let $S=(s_1,s_2,...s_k...)$ be a sequence of positive integers and, consider the set P(S) consisting of all numbers which are representable as a sum of a finite number of distinct terms of S. We say that S is complete if all sufficiently large integers belong to P(S). For a complete sequence, we call the largest integer not in P(S) the threshold of completeness T(S). It is known that for the sequence of squares

S=(1,4,9,16,..) T(S) - 128

and for the sequence of cubes

S=(1,8,27,64..) T(S) = 12758

■ **PROBLEM SL.** Determine the value of T(S) for the sequence of fourth primes and triangular numbers. (Generated using n(n + 1)/2).

Report on "Chiefs and Indians" (*Numbers Count, PCW* April 1996)

"Stop Press": Rex Gooch analysed up to six consecutive prime pairs to 109 and also confirmed Nigel Backhouse's result, of 14 consecutive prime pairs starting at 678771479, 678771481, while John Sutton looked at the alternative problem of the span containing n prime pairs, relaxing the requirement of no intervening primes. A future research area?

Now to the "Chiefs and Indians". Alan Cox quotes from Rouse Ball where the "Josephus problem" is referred to with the reference Hegesippus's "De Bello Judaico". Nigel Hodges generates samples of the numbers of Indians needed for the Chief to be successful for "step-factors" up to 49. For example, 1169262 Indians will constitute good news for the Chief if the "step-factor" is 44.

However, the worthy prizewinner this month is Robert Newmark of Cleadon, Sunderland, who programmed in C on a Toshiba T2110-486DX for up to 5,000 Indians with jumps from two to 20: total analysis in one second. The program is available on request.

•PCW Contributions Welcome

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles. Write to him at **22 Gors Fach, Pwll-Trap, St. Clears, SA33 4AQ or phone 01994 231121**.