Not numerology but numeralogy!

There's a world of difference between the o and the a, as Mike Mudge explains.

Numerology is variously defined as the study of numbers as supposed to show future events or the relationship between numbers and the occult. However, the term *numeralogy*, supplied by P Castini of Arizona, USA, is defined (by him) as "Properties of the Numbers": his proposal for a Numbers Count column includes some 37 sequences each with a rule of generation and some associated queries for investigation.

There follows a (random?) sample of these. Others may be included at a later date depending on the popularity of such research areas.

The **PROBLEM CAS. (n).** is the same in every case, *viz.* implement a computer algorithm to generate the defined sequence and hence, or otherwise, investigate the associated queries.

S(1). Non-arithmetic Progression. General definition: If m_1 & m_2 are the first two terms of the sequence, then m_k for k greater than 2 is the smaller number such that no 3-term arithmetic progression is in the sequence, i.e. we do not find

$m_p - m_q = m_q - m_r$

for distinct p,q & r.

e.g. if

`m1= 1 & m2 = 2`

we generate

`1,2,4,5,10,11,13,14,28,29,31,...`

Generalised S(1) Same initial conditions, but no t-term arithmetic progression in the sequence for t greater than 3.

Query How does the density of such a sequence, i.e. the fraction of the integers less than N which it contains, vary with N, (m_1,m_2) & t?

S(2). Prime-product sequence Here T_n is one greater than the product of the first n primes with the proviso that $T_1=2$. Sequence begins

`2,7,31,211,2311,30031...`

since 2*3*7*11*13+1 = 30031.

Query How many members of this sequence are prime numbers?

S(3). Square-product sequence As S(2) above with primes replaced by squares, viz.

`2,5,37,577,14401,`
`518401,....`

since

$1^2*2^2*3^2*4^2*5^2*6^2 + 1 = 518401$

Query How many members of this sequence are prime numbers?

Generalised S (3) Replace squares by cubes, fourth powers, etc. and investigate the same query. May also be generalised using the products of the factorial numbers

`1,2,6,24,120,720....`

Now let (T_n) be a sequence defined by a property P and screen this sequence, selecting only those terms whose individual digits hold the property P to obtain the S. P-digital subsequence. e.g. the S. square-digital subsequence

`0,1,4,9,49,100,144,..`

is obtained from

`0,1,4,9,16,25,36,49,..`

by selecting the terms whose digits are all perfect squares — only 0,1,4 & 9 allowed.

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Numbers Count, June 1996

"Sequence of events", Descriptive Number Sequences Part (1), *PCW* June 1996, proved very popular. It is intended to review at length the two parts of this topic in the next issue. Suffice it to announce the prizewinner as Jean Flower of The Mathematics Centre, Chichester IHE, Upper Bognor Road, Bognor Regis, West Sussex PO21 1HR, who used Mathematica on a Pentium 120 and (eventually) was able to find all cycles of length less than 17, with a greater than 1 and n greater than 13. All of this was accompished in about five minutes of processor time and was accompanied by a fascinating alphabetic version of the same problem. Consider the sequence of sentences. "This sentence contains three hundred and seventeen occurences of the letter 'e'", the next term being a sentence which decribes the previous one etc. What about carrying this analysis on a computer?

More to come on this topic.

Similarly for the S. cube-digital subsequence and higher powers.

S (4). Consider the S. prime-digital subsequence

```
2,3,5,7,37,53,73,..
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Query Is this sequence infinite?

S (5). The S. odd sequence

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1,13,135,1357,13579, 1357911,...
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Query How many terms are prime?

S (6). The S. even sequence

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2,24,246,2468,246810,..
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Query How many terms are the nth powers of a positive integer?

S (7) The S. prime sequence

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2,23,235,2357,235711,...
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Query How many terms are prime?

For further study of S(4) through (7) see: Sylvester Smith, *Bulletin of Pure and Applied Sciences*, vol. 15. E (no. 1) 1996. pp101-107. A set of conjectures on Smarandache* Sequences.

**All the sequences discussed this month have appeared in print under Smarandache Notions.*

For further information on this area of work see Smarandache Notions Journal, vol. 7 no. 1-2-3, August 1996. ISSN 1084-2810. Department of Mathematics, University of Craiova, Romania.

Something totally different

Eric Adler has drawn my attention to the approximate sizes of elements in the Mathematica 3.0 Software Package where "Front end etc. 6.0Mb, Kernel etc. 18,5Mb, MathLink Libraries 0.5Mb and Fonts 4.5Mb total 27.5Mb whilst Standard Add-on Packages at 9.0Mb together with The Mathematica Book of 36Mb, Listing of Built-In Functions at 5.5Mb, Standard Add-on Packages occupying 11.0Mb and Additional Documentation of 15.0Mb (the latter four items totalling 66Mb) yield 74.5Mb. The total size of storage (again approximate) is quoted as 96Mb whilst strict addition yields 106.0Mb."

Eric asks: "How do they get that?" and offers ten IBM format 3.5in 1.44Mb floppy disks as first prize, with 40 IBM-format 3.5in 1.44Mb floppy disks with UBASIC as runners-up prizes. Facetious answers such as "They used a Microsoft Calculator" or "They are measuring using Microsoft Drive Space" will not be eligible for the first prize!

Stop press!

Would Duncan Moore please let me have his address as I have some information for him. Sorry, Duncan, for the inefficiency of my filing system!

Following on from the study of "Golomb rulers" in the August 1996 issue of *PCW*, at least one reader has expressed an interest in the "Circular Golomb Ruler". Here, the problem is essentially the same except that the points are spaced around the circumference of a circle and distances measured along the circumference also. Apparently solutions are known for some n (maximum distance to be measured); it is further known that for certain n, no solution is possible. What happens if the distance is measured in a straight line!?

Any investigations of this month's queries may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthenshire SA33 4AQ, tel. 01994 231121, to arrive by 1st May 1997. All material received will be judged using suitable criteria and a prize will be awarded by *PCW* to the best entry (SAE for return of entries, please).

• PCW Contributions Welcome

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles. Email him at **numbers@pcw.vnu.co.uk**