Mods and rockers

Mike Mudge JAMS with mod sequences. No, he hasn't joined a retro band; here he presents a stimulating exercise in occurrences to get your feet tapping and your calculators clicking.

JAMS, or Jonathon Ayres Mod Sequences, are believed to have their origins in Leeds in the autumn of 1996. I am indebted to Jonathon for the following presentation of the idea which both he, and I hope readers of this column, will find interesting and stimulating.

Mod sequences

The mod sequence is defined as
X(n) = (2*X(n-1)+1) mod n
where n starts at 1 and x(0) equals 0. The first few numbers in the mod sequence are 0, 1, 0, 1, 3, 1, 3, 7, 6 and 3.

1. Occurrence of X

When does a number occur in this sequence? The first occurrence of the numbers 0 to 19 in the mod sequence are shown in Fig 1.

All numbers less than 1,000 occur in this sequence, for n less than 10,000,000, with the exception of 204, 344, 614, 622, 876 and 964. These first occur at:

X(n)= 614,	n= 10629529
X(n)= 204,	n= 15245143
X(n)= 344,	n= 26713415
X(n)= 622	n= 47286732
X(n)= 964	n= 67815823

I have not been able to find the first occurrence of X(n)=876, but if it does occur n is bigger than 75,000,000.

2. Special values of X(n)

- X(n)=0 for n = 1, 3, 79, 35, 431, 1503, 2943, 6059, 6619, 18911 and 54223.
- X(n)=n-1, for n=1, 2, 8, 32, 46, 392, 12230, 155942, 659488, 1025582, 10471228 and 3437088
- X(n)=n/2, for n=2, 78, 234, 430, 1502, 2942, 6058, 6618, 18910 and 54222
- X(n) and n end in the same last four digits for n=34875, 52363, 54975 and four others less than 100,000, and with the last five digits of both the same, the only values of n less than 1,000,000 are n=389103, 469599 and 742955.

3. Distribution of X(n)

- The most common occurring values of X(n) are of the form 2^p-1, so that for n less than 1,000,000, the number 63 occurs 47 times.
- The average value of x(n) is about n/4.
- There are no values of n greater than 1 so that X(n)=X(n+1), but for X(n)=X(n+2) this is true for n=6, 7, 12, 13, 24, 25, 174, 175, 2448, 2449, 3072, 3073, 6768 and 6769.
- X(n)+1=X(n+1) is true for the values of n, n=3, 5, 81, 237, 433, 1505, 2945…

Fig 2 shows the first values of n so that X(n)+a = X(n+1). All values of **a**, less than 500, occur for **n** less than 10,000,000 except for 205, 215, 345 and 391.

- For pairs of numbers x and y, y is at most 2x+1. The values of x where y has values other than 2x+1, are x=1,3,6,7,13,14,15, 16,17,18,20,23...

Fig 1

First occurrences of the numbers Y, Y=0 to 19, so that X(N)=Y

Y	N	Y	N
0	0	10	149
1	2	11	27
2	53	12	91
3	5	13	18
4	71	14	21
5	26	15	17
6	9	16	43
7	8	17	20
8	19	18	29
9	72	19	50

Fig 2

First values of n so that X(n)+a = X(n+1)

A	X	A	X
1	3	11	151
2	6	12	29
3	55	13	93
4	9	14	64
5	73	15	29823
6	28	16	33
7	63	17	45
8	18	18	42
9	21	19	71
10	74	20	52

Supernumerary

- On 6th December, Tony Forbes of Kingston-Upon-Thames announced his discovery of a triplet of 1,083-digit primes, believed to be the largest known prime triplet. *(Further details of these numbers and the underlying theory/computation on request — MM.)*
- Anyone who knows the means of obtaining a "zooming Mandlebrot plotter" please email Gogul@aol.com.
- Anyone wishing to get involved in the "Great Internet Mersenne Prime Search" mentioned in *PCW* (Jan) should contact Nigel Backhouse, Division of Applied Mathematics, University of Liverpool, M&O Building, Liverpool L69 3BX (*Kevin Edge, please note).*
- In response to frequent requests for reasonably priced (or free) software for long integer manipulation: I can provide UBASIC free of charge on receipt of a suitably stamped, addressed, padded bag.

Numbers Count (*PCW*, September '96) — 'Fraction Action'

- Gareth Suggett obtained successive length records for the period of the continued fractions of the square roots of the non-square integers up to d=10,000, terminating with d=9,949 having cycle length 217. However, Gareth discovered a program called "CALC", written by KR Matthews of the University of Queensland. The MSDOS version is available from the Mathematics Archives ftp site: ftp://archives.math.utk.edu/software/msdos/number.theory/krm-calc. On a 25MHz 386 PC, each of the 10-digit results quoted in the original article can be obtained in about 20 minutes. The final 11-digit result was confirmed on a 133MHz Pentium in 15 minutes, producing a 6.8Mb output file!

John Borland observed that at some time, "continued fractions were a standard topic in higher mathematics". Readers' experiences of instruction in this topic would be most interesting, together with their personally recommended reference books both for numerical and function approximation theory applications.

This month's prizewinner, however, is Duncan Moore of Birkenhead for his major contribution to "Something Different", spread over August 1993 and January 1997. The total number of solutions now known is 30.

Also in relation to this problem, Henry Ibstedt reported *(November '96)* finding one with three of p, q, r, s, t sharing one factor and the other two sharing a different factor. This solution is p=286, q=154 sharing the factor 2, and r=s=t=11 sharing the factor 11 with (2, 11) = 1.

Henry points out that p and q also share the factor 11 but that this was not excluded from the question — there is still a great deal of work to be done before this problem is fully understood.

Questions

- Do all numbers occur in this sequence, and also, do they occur an infinite number of times?
- Is there always a value of n, for every a (positive or negative) so that X(n)+a = X(n+1)?
- Is there a way of predicting when a number will occur in the sequence?
- Is there a formula which gives the n'th value of the sequence, without calculating the rest of the series?
- What happens for other sequences, such as x(n)=ax(n-1)+b mod n or x(n)=(x(n-1) +x(n-2)) mod n?

Something different

This item was taken from *Computer Weekly* (19th January edition, 1989).

Following up on the observation that $15226_{10} = 62251_7$ and further that $99481_{10} = 18499_{16}$ (where the subscript denotes the base in which the number is represented), find the lowest five-digit number (in any base). Generalise this process to n-digit integers.

Answering back…

Please send any investigations of the above problems to Mike Mudge at 22 Gors Fach, Pwll-Trap, St Clears, Carmarthenshire, SA33 4AQ (tel 01994 231121), to arrive by 1st July, 1997. All material received will be judged according to suitable criteria and a prize will be awarded by *PCW* to the best entry arriving by the closing date (an SAE is required for the return of entries). Each contribution should contain brief descriptions of the hardware and coding used, together with run times and a summary of the results obtained.

General comments on the topic of JAMS would be welcome, together with any practical (or unusual) applications of integer arithmetic in number bases other than 2 and 10.

PCW Contact

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational maths, together with suggested subject areas or specific problems for future articles. Email numbers@pcw.vnu.co.uk