What comes next?

Mike Mudge reads from the Book of Numbers and throws down a challenge to readers.

The LOOK and SAY Sequence appears in *The Book of Numbers* by John H Conway (of *The Life Game* fame) and Richard K Guy, ISBN 0-387-97993-X, Springer-Verlag, 1996.

Consider the sequence

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1, 11, 21, 1211, 111221, 312211,
13112221, 1113213211,
31131211131221...
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Have you guessed the general rule? The first term is one "one" so the second is "one one". This consists of two "ones" so say the third is "two one". This in turn is seen as one "two" and one "one" and so say for the next term "one two one one", and so on.

The question is, how many digits are there in the n^{th} of this sequence? Now using those epic words, it can be shown that the number of digits in the n^{th} term is roughly proportional to (1.3035772...)n, where Conway and Guy quote this constant to 49 decimal places followed by the 71st degree polynomial equation with integer coefficients less than 20, of which it is a root. I shall be amazed if any *PCW* readers can reproduce this result, but the associated Numbers Count problem is as follows.

L&S Sequence

Design and implement an algorithm to generate consecutive terms of the LOOK and SAY Sequence up to the available system limitation; fit x^n to the length of the n^{th} term by any empirical method. Is the resulting x anything like the above constant?

Introducing the Pseudo-Smarandache Function

This concept is due to Kenichiro Kashihara (private communication).

As a first step, readers are asked to recall the definition of the Smarandache Function S(n), as the smallest integer m such that n evenly divides m! (factorial m = m! = 1.23..m), for any integer n greater than or equal to 1. The Pseudo-Smarandache Function has a similar definition where the multiplication in the definition of the factorial function is replaced by addition — it is denoted by Z(n).

n	1	5	10	15	20	25	30	35
Z(n)	1	4	4	5	15	24	15	14

Problem: Z(n)

(a) Find all values of n such that:

(a) Z(n) = z(n+1)

(b) Z(n) divides Z(n+1)

(c) Z(n+1) divides Z(n).

(b) Is there an algorithm that can solve:

(a) Z(n) + Z (n+1) = Z (n+2)

(b) Z(n) = Z(n+1) + Z(n+2)

(c) Z(n) *Z(n+1) = Z(n+2)

(d) Z(n) = Z(n+1) * Z(n+2)

(e) 2*Z(n+1) = Z(n) + Z(n+2)

(f) Z(n+1)*Z(n+1)=Z(n)-Z(n+2)

(c) For a given natural number M how many solutions are there to Z(x) = m?

(d) Are there any instances where FOUR CONSECUTIVE POSITIVE integers yield a monotonic (increasing or decreasing) sequence of Z-values?

Harmonic Numbers

The n^{th} harmonic number Hn is defined as the sum of the first n terms of the harmonic series, thus Hn 1 + 1/2 + 1/3 + 1/4 + 1/5 ... +1/n.

Problem Harmonic

Design and implement an algorithm for generating harmonic numbers. Use it to discuss that: "the n^{th} harmonic number is about one n^{th} of the n^{th} prime number."

Investigations of the above problems may be sent to Mike Mudge — the address is in the box alongside: a prize to the best entry received by 1 December 1997. Please include an SAE if you want your entry back.

Review of Numbers Count 164, December 1996, *Close Relations*

(Correction — Page 294, column 3, line -1: *Scientific American* should, in fact, read *American Scientist*.)

Jim Parker jim@jebp.demon.co.uk (address on request) has sent a late submission, 2 June 1997, advising of a program operating in a particular form of arithmetic which provides the full results of the snub dodecahedron. Some knowledge of abstract algebra (field theory) is needed to fully appreciate this work.

John Sharp saw recurrence relation $T_n = 2T_{n-1} - T_{n-4}$ associated with $t^4 = 2t^3 + 1$ (number E) yields: 0,1,1,1,2,3,5,9,16,29,... where ratio of successive terms converges, very slowly, to the Tribonacci Number.

Duncan Moore, Nigel Hodges and others found simple algebraic functions for A through I and partial results for the snub-dodecahedron, e.g.

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I = (t(T+2))**1/2,
G = (2t+3)**1/2
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Nigel succeeded in proving, in relation to problem SL, that: T(2) = 128, T(cubes) = 12758, T(fourth powers) = 5134240, T (fifth powers) = 67898771, while T(sixth powers) greater than 500 million while T(triangular numbers) = 33.

After much heart-searching, the very worthy winner is Paul Richter of Flat 2, 21 Queens Road, Tunbridge Wells, Kent TN4 9LL, for a non-sophisticated approach to this investigation. Details can be obtained from Mike Mudge, or John Sharp at The Glebe, Watford WD2 6LR.

PCW Contacts

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational maths, together with suggested subject areas or specific problems for future articles. Email numbers@pcw.co.uk.